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ECE
PM 1 (B).

Ace Academy

Control Systems
**Control Systems**

- Books:
  1. Control System → NISE
  2. Control System → Nagrath & Gopal
  3. Automatic Control System → B.C. Kuo.
  5. Modern CS → Ogata.

- Topics:
  - TF, BD, SFA → 1M (or) 2M
  - TDA
    - Transient Analysis
    - Steady State Analysis → 2M
  - Stability → Time Domain Tech. → RH | RL
  - Frequency Domain Tech. → BP | NP
  - Compensators | Controllers
  - State Space Analysis → 2M
\[ \text{Introduction:} \]

* **Toussner, Block Diagram & SFG:**

\[ \Rightarrow \text{Toussner function is a mathematical equivalent model for the system.} \]

\[ \Rightarrow \quad \text{TF} = \frac{1}{s+1} \]

\[ \downarrow \]

No. of Storage elements

\[ \downarrow \]

No. of Time Constant

\[ \Rightarrow \quad \text{Objective of the CS: Set desired accurate output.} \]

\[ \Rightarrow \quad \frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} \quad \text{RC LPF} \]

\[ \Rightarrow \quad \frac{V_o(s)}{V_i(s)} = \frac{1}{sC + 1} \]

Let \( \tau = RC \).

\[ \therefore \quad \frac{V_o(s)}{V_i(s)} = \frac{1}{s\tau + 1} \Rightarrow \frac{C(s)}{R(s)} = \left( \frac{1}{s+1} \right) \]

i.e. \[ \frac{C(s)}{R(s)} = \frac{1}{s+1} \quad \text{for LPF.} \]
\[ \text{Why LPF} \]

\[ \text{Noise} \rightarrow \text{Eliminated} \rightarrow \text{LPF} \]

\( 1 \) Noise get eliminated by LPF.

\( 2 \) Components are more stable at LF.

\[ \Rightarrow \text{System: } = \frac{K (1 + s \tau_1) (1 + s \tau_2) \ldots}{s^n (1 + s \tau_n) (1 + s \tau_b) \ldots} \]

\[ \Rightarrow \text{Always selecting:} \]

\( 0 \) Poles > Zeros \( \Rightarrow \) For LPF

\( \Rightarrow \) Then only it is 

\[ \text{Stoically Proper TF} \]

\( 0 \) When Poles = Zeros

\( \Rightarrow \) it act as 

\[ \left\{ \begin{array}{c} \text{LP} \\ \text{HP} \\ \text{BP} \\ \text{BS} \end{array} \right\} \rightarrow \text{Proper TF.} \]

\( \Rightarrow \) Zero at origin is not acceptable.

\( 0 \) When Poles < Zeros \( \Rightarrow \) Improper TF. \( \Rightarrow \) HPF.

\( \Rightarrow \) BD, SPo \( \Rightarrow \) To find the overall TF of the System.
Time Domain Analysis:

- The objective of the TDA is used to evaluate the performance of the system w.r.t. time.

\[ \gamma(t) \rightarrow \text{MEM (or) T.F. (or) System} \rightarrow c(t) \]

- More accurate if less relative stable and more oscillatory.
- Less accurate if more relative stable and less oscillatory.

- \( t_0, \tau, t_s, t_p \): \( t_0 \) - Slow Res.
- \( M_p, \theta_s \): More accurate
- \( t_0, \tau, t_s, t_p, M_p, \theta_s \): \( t_0 \) - More accurate

- \( u(t) \rightarrow y(t) \)
- \( y(t) \rightarrow \gamma(t) \)
- \( y(t) \rightarrow \gamma(t) \)
- \( y(t) \rightarrow \gamma(t) \)

- Fourier domain analysis is used to find \( \omega_m \) and \( \phi_m \).
Control System Specification:

- Speed → $t_0$, $t_s$ \(\downarrow\downarrow\) → Quick Res.

- Accuracy → $e_{ss}$ ↓ → Small → More Accurate.

- Stability → $GM$ & $PM$
  - \[\uparrow\uparrow\]
  - Mod. R.S. (Adv.)
  - Slow R.S. (dis.)

- Optimum
  - Value of $GM$ → 5 dB to 10 dB
  - $PM$ → 30° to 40°.

- ToA Should be insensitive to Unwanted Parameters such as Temperature, Noise & Disturbance.

- $e_{ss}$: Steady State error.
- $M_p$: Peak overshoot.
- $t_d$: Delay time.
- $t_r$: Rise time.
- $t_p$: Propagation time.
- $t_s$: Setting time.
Stability

Closed loop system.

OL System

Closed loop system

Always defined in terms of OL TF.

\[ G(s), H(s) \]

\[ G(s) \times H(s) = 1 \]

OL TF of a non-unity FB sys.

\[ \frac{G(s)}{R(s)} = C(s) \]

CL TF

OL TF of a unity FB sys.

\[ \frac{C(s)}{R(s)} = \frac{C(s)}{1+G(s)} \]

OL TF of a unity FB sys is

\[ G(s) = \frac{10}{s-8} \]

Then system is ___.

Ans:

\[ CL TF = \frac{C(s)}{1+G(s)} = \frac{10}{s+2} \]

So, Stable.

Why there is no need of stability technique for OL system?

\[ G(s) = \frac{S+1}{s^2 (s+2) (s+3)} \]

Poles and zeros location are identified directly from \( G(s) \).
\( s \) \[ \text{CLTF of a unity FB system is,} \]
\[
G(s) = \frac{s+1}{s^2(s+2)(s+3)}
\]
\[
\text{CLTF} = \frac{G(s)}{1 + G(s)} = \frac{s+1}{s^4 + 5s^3 + 6s^2 + s + 1}
\]

\( \Rightarrow \) The Feedback changes the locations of the poles. Identification of new location of the poles are very difficult, hence we need a stability technique for closed loop stability.

\( \Rightarrow \) Stability Technique:

1. **Nyquist** \( \rightarrow \) No. of poles on RL, Range of \( k \), \( \text{gm} \& \text{pm} \) can be find.

2. **RL** \( \rightarrow \) Nature of the system.

3. **BP** \( \rightarrow \) \( \text{gm} \& \text{pm} \).

4. **RH**.

\( \Rightarrow \) The T-O technique gives the transient analysis and steady state (ss) Analysis.

\( \Rightarrow \) The F-O technique gives the only steady state (ss) Analysis.

\( \Rightarrow \) The Stability Analysis is a steady state Analysis.
Transportation Delay Lag System.

\[ \mathcal{L} \left[ g(t - \tau) \right] = e^{\tau \frac{\sigma}{s}} \cos(s) \]

delay

**TPA:**
\[ e^{\tau s} \Rightarrow (1 - \tau s) + \frac{(\tau s)^2}{2!} + \cdots + \infty \text{ poles.} \]

\[ \Rightarrow \text{So, TPA Not gives accurate stability.} \]

**FDA:**

\[ \begin{align*}
B \rightarrow M & \Rightarrow \omega \\
\varphi & \Rightarrow \omega
\end{align*} \]

\[ P \rightarrow M \rightarrow 0 \]

\[ e^{-\tau s} = e^{-j\omega}, \quad m = \tau \quad \Rightarrow \varphi = -\omega \tau. \]

In FDA there is no any approximation.

* Compensators Controllers:

\( \Rightarrow \) It is required to get desired system.

It is a simple electrical NLC which adds the poles and the zeros to the system in order to get the desired performance of the system.

* Steady State Analysis:

\( \Rightarrow \) It is only valid for non-linear, linear, time variant & time invariant system. It is define for dynamic system.
A system is a group of elements (or a physical component) arranged in such a way that it gives the proper output to the given input.

A proper output is may (or) may not be the desired output.

For example:

- A FAN w/o Blades ⇒ Not a System
  ⇒ No proper olp ⇒ No air flow.

- A FAN w/o Regulator ⇒ System
  ⇒ Proper olp ⇒ Air flow.

- A FAN with Regulator ⇒ Controlled System
  ⇒ Gives Desired olp.
A control system is a group of physical components arranged in a such a way that it gives the desired output by means of control, (or) regulate (or) Command either direct (or) indirect method to the given input.

Control systems are classified in two ways based on controlling action.

i) open loop control system (OLS).

ii) closed loop control system (CLS).

Open Loop Control System: (manual):

```
Reb. ilp
Controllers --> Process
Manually Operated Switch
```

Reb. ilp is nothing but a desired olp. [what we required or what we need].

A system in which the controller action is completely independent of the output of the system is called open loop control system. E.g. Fan, lights, cooler, traffic light.
A system in which the controller action depends on the olp then it is called **closed loop control system**.

- **e.g.**: Ac, Refrigerators, Human beings, Automatic iron box and so on.

Any system which is having a sensor and provision to select the reference olp.

**Feedback Network**:

- It is a property of the closed loop system which brings the olp to i/p and compared with set i/p so that appropriate control action formed to make the error equal to zero.

- Error equal to zero, the system is stable which give the desired olp.
The main components in F1B now is R, L, C. The maximum gain of F1B now ratio is 1.

The best F1B is unity (-ve) F1B. Because the (-ve) F1B improves the relative stability. (loop gain > 0).

The steady-state errors are valid for only unity F1B system. If non-unity F1B system is given it should be converted into unity F1B.

The F1B now may consist the transducers which converts the energy from one form to another form.

**Transfer Function:**

The transfer function is basically mathematical equivalent model for the system.

The order of the transfer function represents the no. of storage elements (or) no. of the time constants.

Note: Whenever some kind of elements connected either series (or) parallel
Components.

For e.g.

\[ \text{2nd order:} \]

⇒ First definition of T.F.:

⇒ A T.F. of a linear time-invariant system is defined as the ratio of Laplace transform of output to the Laplace transform of input with all initial conditions are zero.

\[ \text{T.F.} = \left. \frac{L \{ y(t) \}}{L \{ x(t) \}} \right|_{t=0} \]

⇒ LTI system:

⇒ The LTI system is nothing but RLC circuit because the RLC components give the linear transfer characteristics and the R, L, C components values are not changes w.r.t. time.

⇒ In TF analysis, the initial conditions must be zero because the output should not depend on the past history of the
System. It should depend on the component values and present IIP.

⇒ Second Term of T.F.:

⇒ The T.F of the LTI system is defined as Laplace transform of impulse response with all initial conditions are zero.

\[ \text{T.F.} = \frac{L[\text{Impulse Response}]}{L[\text{IIP}]} \bigg|_{I_i=0} \]

\[ = \frac{L[\text{Impulse}]}{L[\text{IIP}]} \]

\[ = \frac{1}{s+1} \]

(\because L[\delta(t)] = 1)

⇒ \[ \text{Impulse} \rightarrow \frac{1}{s+1} \rightarrow \text{Impulse Response} \]

\[ \frac{C(s)}{R(s)} = \frac{1}{s+1} \]

\[ R(s) = 1 \]

\[ \therefore C(s) = \frac{1}{s+1} \cdot 1 \]

\[ \text{System Comp.} \]

\[ \text{So, called sys. res.} \]

⇒ if we take \( R(s) = \frac{1}{s} \), i.e. \( \delta(t) = u(t) \):

\[ \therefore C(s) = \frac{1}{s+1} \cdot \frac{1}{s} \]

\[ \text{Response has input Gauss terms} \]

\[ \text{so it is not called sys. response.} \]

\[ \therefore \]
The impulse response gives the system behaviour (or) system characteristics because the impulse response consists of only system parameters. No & system presents in the impulse response. Hence the impulse response is called system response (or) natural response (or) free forced response.

If the signals are unit step, ramp (or) parabolic then their response is called forced response.

Transfer Function to Electrical NW:
Any system basically defined in terms of Bode.

The standard form of the system is described as

\[ G(s) = \frac{K (1 + \gamma_1) (1 + \gamma_2) \cdots}{s^n (1 + \gamma_a) (1 + \gamma_b) \cdots} \]

K & \gamma are called system parameters.
K: System Gain.
\gamma: System Time Constant.

n: Type-n system.
Find the System Gain, Type and Order to the following system.

\[
\frac{C(s)}{R(s)} = \frac{10(s+5)^2}{s^3(s+2)(s+10)} \quad \text{[Pole-zero form]}
\]

**Solution:**

\[
\frac{C(s)}{R(s)} = \frac{10 \times 25 (1 + \frac{s}{5})^2}{4 \times 10 \times s^3 \left( \frac{s}{2} + 1 \right)^2 \left( 1 + \frac{s}{10} \right)}
\]

\[
\frac{C(s)}{R(s)} = \frac{6.25 (1 + \frac{s}{5})^2}{s^3 \left( 1 + \frac{s}{2} \right)^2 \left( 1 + \frac{s}{10} \right)} \quad \text{[Time constant form or standard form]}
\]

Type: \( \Rightarrow 3 \)  
Order: \( \Rightarrow 6 \)  
Gain: \( \Rightarrow 6.25 \)

Find the Type and Order to the given CLTF of a unity feedback system.

\[
\frac{C(s)}{R(s)} = \frac{2s+5}{s^5 + 4s^4 + 6s^3 + 7s^2 + 2s + 5} = \frac{C(s)}{1+G(s)}
\]

**Solution:**

Here, \( \frac{C(s)}{R(s)} = \frac{C(s)}{1+G(s)} \)

\[
\frac{C(s)}{1+G(s)} = \frac{2s+5}{s^5 + 4s^4 + 6s^3 + 7s^2 + 2s + 5}
\]
\[
C(s) = \frac{s^5 + 4s^4 + 6s^3 + 7s^2 + 2s + 5}{s^5 + 4s^4 + 6s^3 + 7s^2}.
\]

\[
C(s) = \frac{2s + 5}{s^2 (s^3 + 4s^2 + 6s + 7)}.
\]

So, order \( \rightarrow 5 \).

Type \( \rightarrow 2 \).

\textbf{Note:}

\( \Rightarrow \) The Type & order is not defined for closed loop TF. To get a Type and order for CL sys. require OLT or unity feedback system.

\textbf{Characteristic Equation:}

\[
\Rightarrow \frac{C(s)}{R(s)} = \frac{(s-10)}{(s+1)(s+5)}.
\]

\[
= \frac{K_1}{s+1} + \frac{K_2}{s+5}.
\]

\[
\Rightarrow \frac{C(s)}{R(s)} = K_1 e^{-t} + K_2 e^{-5t}.
\]

\[
C(s) = \frac{K_1}{s} + K_2 = \frac{K_1}{s} + \frac{K_2}{s}.
\]
Denominator terms decide the character of the system not numerator terms. So for chaotic e.m we take denominator term equal to zero.

The denominator of transfer function makes equal to zero then it is called characteristic equation.

The chaotic e.m gives the system behaviour (or) characteristics of the system.

For a CL system, the characteristic equation is

\[ 1 + G_c(s) \cdot H(s) = 0. \]

The roots of chaotic e.m is called poles.

Pole:

The pole is nothing but the negative of inverse of system time constant and which magnitude of the TF becomes \( \infty \).

\[ S_p = -\frac{1}{T_a}, -\frac{1}{T_b}, \ldots \quad \text{TF} \rightarrow \infty. \]
* Zero:

\[ S_2 = -\frac{1}{\tau_1}, -\frac{1}{\tau_2}, \ldots \]  

\[ |TF| = 0. \]

\[ \text{H.B.} \]

\[ \Rightarrow \text{The pole can affect the system response and system stability, but not the zero.} \]

* Time Constant:

\[ \Rightarrow \text{The time constant gives the system behaviour. If the time constant is very very large then it is called slow response system. Because it takes large time to reach the steady state.} \]

\[ \Rightarrow \text{Practically any time system takes the } 5\tau \text{ to reach the steady state.} \]

\[ \gamma = -\frac{1}{\tau} \]

\[ \text{Real part of the dominant pole} \]

\[ \text{H.B.} = \]
Dominant Pole: \( \frac{1}{s} \)

The pole which is very close to the imaginary axis is called a dominant pole.

1. Find the equivalent 1st order system.
2. Find the system time constant for:

\[
\frac{C(s)}{R(s)} = \frac{1}{(s+1)(s+10)}
\]

Solution:

\[
\tau = \frac{-1}{-10} = 0.1 \text{ sec}
\]

Insignificant pole has less time constant, so good performance and hence best pole.

Dominant pole has large time constant, so bad pole. It affects the system.

So we have to compensate it by adding zero at same position, so that we discuss only for DP.
Insignificant Pole $\leq 5\times$ times of Dominant Pole.

Then only it is called insignificant pole.

\[ -10 \leq s \leq 0 \]

\[ \Rightarrow -10 < s \]

\[ \text{Ins}(\gamma) \leq \frac{Dp(\gamma)}{5} \]

\[ \text{Ins}(\gamma) \leq \left( \frac{1}{5} = 0.2 \right) \]

* Insignificant Pole:

→ The poles which lies in the left most side.

→ The insignificant Pole time constant must be less than \( \alpha \) equal to 5 times or the dominant pole time constant.

That means insignificant pole.

\[ \text{Isp}(\gamma) \leq \frac{Dp(\gamma)}{5} \]

\[ \text{H.8} \]

→ The best pole is the insignificant pole.
because it gives the very quick response and more relatively stable. Because of the dominant pole the system response become the slow and the system becomes less relatively stable.

⇒ The insignificant poles are neglected because even if insignificant poles are neglected there is no much change in the system response.

⇒ \[ \frac{C(s)}{R(s)} = \frac{1}{(s+1)(s+10)} \]

System Response \[ R(s) = 1 \]

⇒ \[ C(s) = \frac{1}{(s+1)(s+10)} \]

⇒ \[ C(s) = \frac{1}{3(s+1)} - \frac{1}{9(s+10)} \]

\[ \text{ILT} \rightarrow C(t) = \frac{1}{3} e^{-t} - \frac{1}{9} e^{-10t} \]

⇒ \[ T = 15 \text{ op res. ISP Res. } T = 0.1 \text{sec.} \]

⇒ \[ \mathcal{L} \left[ e^{-at} \right] = \frac{1}{s+a} \]

⇒ \[ \mathcal{L} \left[ \sin(bt) \cos(bt) \right] = \frac{b \cos s}{s^2 + b^2} \]
\[
\Rightarrow \mathcal{L} \left[ e^{-at} \sin bt \right] = jwb \quad \text{S-plane}
\]

\[
\Rightarrow \mathcal{L} \left[ t^n e^{-at} \right] \Rightarrow \text{(nt+1) poles are repeated.}
\]

\[
\Rightarrow \text{In the response, exponential powers are.}
\]

Real part of the poles, sine or cos function are imaginary. Part of poles.

The \( t \) term represents the repeated nature of the poles.

\[
\Rightarrow \text{To get the system time constant from the response, compare the response with } e^{-t/\tau}.
\]

\[
\Rightarrow \text{The system time constant is nothing but the dominant pole time constant and it should have the largest value.}
\]

\[
\Rightarrow \mathcal{C}(s) = \frac{1}{(s+1)(s+10)} \quad \text{(pole-zero form). Never neglect pole directly in pole-zero form.}
\]

\[
\text{M.B.}
\]
\[ \frac{C(s)}{R(s)} = \frac{1}{(s+1)(s+10)(s+100)} \]

(a) \[ \frac{1}{s+1} \]
(b) \[ \frac{1}{(s+1)(s+10)} \]
(c) \[ \frac{0.01}{(s+1)(s+10)} \]
(d) \[ \frac{0.001}{(s+1)} \]

Note: The insignificant pole must be neglected only in the time constant form.
While finding system response, system stability, system time constant we consider only poles but not zeros because the system response consist only the poles response terms there is no zeros response term exist in the system response.

\[
\frac{C(s)}{R(s)} = \frac{(s+1)}{(s+2)(s+3)}
\]

Solve: \(R(s) = \frac{(s+1)}{(s+2)(s+3)}\)

\[
C(s) = \frac{(s+1)}{(s+2)(s+3)}
\]

\[
C(s) = \frac{1}{(s+2)} + \frac{2}{(s+3)}
\]

\[
\text{ILT: } C(t) = (-e^{-2t} + 2e^{-3t})
\]
\[ \Rightarrow \text{Stability} \Rightarrow t = \infty \Rightarrow \text{Sys. Res.} \]

\[ \Rightarrow \text{'} \infty \text{'} \text{ value} \Rightarrow \text{Unstable} \]

\[ \Rightarrow \text{Finite value} \Rightarrow \text{Stable.} \]

\[ \text{Stable (\( -\): no pole at RHS).} \]

\[ C(s) = K \frac{\text{CL Zero}}{\text{CL Poles}} \]

\[ \text{CL Sys.} \rightarrow \text{CLTF} = \frac{\text{CL Zero}}{\text{CL Poles}} \]

\[ \text{CL Sys.} \rightarrow C(s) = K \frac{\text{CL Zero}}{\text{CL Poles}} \quad \text{; } k(s) > 1 \]

\[ \text{CLTF} \rightarrow \text{CLTF} = \frac{K \text{ CL Zero}}{\text{CL Poles} + K \text{ CL Zero}} \]

\[ \Rightarrow \text{The CL Zeros never affect the OL Stability.} \]

\[ \Rightarrow \text{The CL Zeros never affect the CL Stability.} \]

\[ \Rightarrow \text{The OL Zeros affect the CL Stability because the CL Poles are nothing but} \]
the sum of the all poles & all zeros with 29
the function of Sys. gain K.

\[ \Rightarrow \text{Note:} \]

(i) To get the OLTF from the CLTF,
subtract numerator in the denominator
when the feedback is unity.

(ii) To get the CLTF from OLTF,
add the numerator in the
 denominator when the F/B is unity.

* Transfer function of the Electrical

\[ \Rightarrow \]

\[ \frac{V_o(s)}{V_i(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)} \]

Impedence across olp
Total CKT impedance

\[ \Rightarrow \]

\[ \frac{V_o(s)}{V_i(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)} \]

\[ \Box \]
Find the TF to the given electrical network and locate the poles in S-plane.

Find the System Response.

\[
\begin{align*}
\frac{V_o(s)}{V_i(s)} &= \frac{1}{sC} \\
&= \frac{1}{1 + s \tau R} \\
\end{align*}
\]

\[
\tau = RC
\]

\[
\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{1}{1 + s \tau}
\]

For SYS. Res. \( V_i(s) = 1 \).

\[
\Rightarrow V_o(s) = \frac{1}{\tau (s + \frac{1}{\tau})}
\]

\[
\text{ILT} \quad V_o(t) = \frac{1}{\tau} e^{-\frac{t}{\tau}}
\]

![Graph showing exponential decay](image)

*Exponential Decay (stable)*
If one or more poles lie in the left of s-plane at different locations then the system response is exponential decay irrespective of position of zeros, the system is stable.

\[ K_1e^{-5t} + K_2e^{-4t} + K_2e^{-2t} + Ke^{-t} \]

Stability:

The movement of the pole in the s-plane is nothing but varying the system components ($R$, $L$, $C$).
Absolutely stable system means the system is stable for all the values of the system parameters (or) system components like ‘k’ from 0 to oo.

Conditional stable system means the system is stable for certain range of system components like ‘k’ from 0 to 100.

Addition of poles & zeros to TF means adding RLC’s components to the system.

The RLC components added to the system in two ways:

1. Series Connection.
2. Parallel Connection.

In series connection the RLC components are added in a forward path.

In parallel connection the RLC components added in a feedback path.
Find the T.F. to the given electrical network and locate the poles in the s-plane by considering $R=0$, $L=1\, H$, $C=1\, F$.

\[ \frac{V_o(s)}{V_i(s)} = \frac{1/SC}{R + SL + 1/SC} = \frac{1}{s^2LC + SCR + 1} \]

\[ \Rightarrow s = 0, \quad L = 1\, H, \quad C = 1\, F. \]

\[ \therefore \frac{V_o(s)}{V_i(s)} = \frac{1}{s^2 + 0 + 1} = \frac{1}{s^2 + 1} \]

Poles: $s^2 + 1 = 0 \Rightarrow s = \pm j$.

\( \Rightarrow \) Non-Repeated Poles on jw axis, System marginally stable.

\[ \Rightarrow V_o(s) = \frac{1}{(s^2 + 1)} \cdot V_i(s). \]

For system response $V_i(s) = \frac{1}{j\omega}$. 

s-plane
\[ V_0(s) = \frac{1}{s^2 + 1} \]

\[ \Rightarrow V_0(t) = \sin t. \]

\[ \text{CaL} \]

\[ = V_0(t) \]

\[ \text{Constant amplitude} \quad \& \quad \text{frec. of oscillation} \]

\[ \text{Undamped oscillation} \quad \text{i.e. Marginally Stable.} \]

\[ \Rightarrow \text{when the poles lies on imaginary axis which are non-repeated then the system response is constant amplitude and f recycled oscillation which are called undamped oscillation.} \]

\[ \Rightarrow \text{Any system which produce undamped oscillation is called undamped undamped system and the system becomes marginally stable.} \]

\[ \text{\textbf{Q1: Repeate the above Problem by Considering}} \quad R = 1 \Omega, \quad C = 1 \text{F}, \quad L = 1 \text{H}. \]

\[ \text{Soln:} \quad \frac{V_o(s)}{V_i(s)} = \frac{1}{s^2 LC + s CR + 1}. \]
\[ \frac{V_o(s)}{V_i(s)} = \frac{1}{s^2 + s + 1} \]

\[ \frac{V_o(s)}{V_i(s)} = \frac{1}{s^2 + s + \frac{1}{4} + \frac{3}{4}} \]

\[ \frac{V_o(s)}{V_i(s)} = \frac{1}{\left(s + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \]

Poles: \[ s = -\frac{1}{2} \pm j \frac{\sqrt{3}}{2} \]

Time Constant
\[ \gamma = \frac{-1}{-\frac{1}{2}} = 2 \text{ sec.} \]

\[ j\omega = \frac{\sqrt{3}}{2} j \]

\[ \omega = \frac{\sqrt{3}}{2} \text{ rad/sec.} \]

Note:

\[ \Rightarrow \text{In Complex Conjugate poles, the real part gives the system time constant and imaginary part gives the freq. of oscillation.} \]

\[ v_o(t) = c(t) \]

\[ = \frac{2}{\sqrt{3}} e^{-\frac{t}{2}} \sin \frac{\sqrt{3}}{2} t \]

\[ c(t) = v_o(t) \]

\[ \gamma = 2 \text{ sec.} \]

\[ \omega = \frac{\sqrt{3}}{2} \text{ rad/sec.} \]
Whenever the poles are complex conjugate in the left of s-plane then the system response is exponentially decay and percent of oscillation which are called damped oscillations. Any system which produce damped oscillations is called under damped system and the system is stable.

Find the system time constant and system response to the given poles location in the s-plane.

\[
\begin{align*}
\text{S-plane} & \\
\text{X} & \\
\text{0} & \\
\text{2} & \\
\text{-jw} & \\
\text{jw} & \\
\end{align*}
\]

Som:

\[
\begin{align*}
\text{S-plane} & \\
\text{X} & \\
\text{0} & \\
\text{2} & \\
\text{-jw} & \\
\text{jw} & \\
\end{align*}
\]

\[
\begin{align*}
\frac{C(s)}{R(s)} &= \frac{(s+3)}{(s+1)(s+2)(s+4)(s-2)} \\
\end{align*}
\]
A given system is unstable and the time constant is not defined. The time constant defined for only stable system.

\[ k_1 \quad k_2 \quad +k \]

\[ \frac{1}{t} \quad +k \quad t \]

\[ C(t) \quad \infty \quad Unstable. \]

\[ K \quad t \]

Note:

\[ \Rightarrow \text{Whenever one (or more) poles lies in the right of the s-plane at different locations on the real-axis then the system is unstable because the system response exponential rises to infinity.} \]

\[ \Rightarrow \text{The system response follows the J pole then the system become stable.} \]
* \( s = -\alpha \)

TF: \( \frac{1}{s + \alpha} \)

Sys. Res: \( 1 \cdot e^{-\alpha t} \)

\( s = 0 \)

\[ \Rightarrow \text{TF} = \frac{1}{s} \]

\[ \Rightarrow C(t) = u(t). \]

* \( s = \alpha \)

TF: \( \frac{1}{s - \alpha} \)

\[ C(t) = 1 \cdot e^{+\alpha t} \]

\[ \Rightarrow \]

\[ C(t) = e^{+\alpha t} \]

\[ \rightarrow \text{Unstable} \]

Exponentially Raised

So, unstable system.
\[ T.F. = \frac{1}{(s+a)^2} \]

\[ C(t) = t \cdot e^{-at} \]

\( t = 0 \Rightarrow 0 \) (term)

\( t = \infty \Rightarrow 0 \) (exp-term)

\[ \Rightarrow \text{for lower value of } t, \text{ } \begin{array}{c} \text{dominant} \\
\end{array} \]

\[ \text{and for higher value of } t, \text{ } \begin{array}{c} \text{dominant} \\
\end{array} \]

\[ c(t) \]

\[ C(t) \]

Unstable.

Repeated pole on jw axis unstable.

\[ \text{Unstable} \]
Complex Conjugate Poles:

\[ TF = \frac{1}{(s+a)^2 + b^2} \]

\[ c(t) = \frac{1}{b} \cdot \text{e}^{-at} \sin bt \]

**Stable**

\[ TF = \frac{1}{s^2 + b^2} \]

\[ c(t) = \frac{1}{b} \cdot \sin bt \]

Undamped
\[
T.F. = \frac{1}{(s-a)^2 + b^2}.
\]

\[
C(t) = \frac{1}{b} e^{-at} \sin bt.
\]

Unstable

\[\Rightarrow S\text{-plane} \quad \text{or TF} \quad \text{Sys. Res.}\]

1. **Real Part** \(\rightarrow\) Exp. term.
2. **Img. Part** \(\rightarrow\) Sine (or) Cos term
   (Cos will come whenever there is a zero at origin)
3. **(Real + Img)** \(\rightarrow\) Product of exp. and sine (or) cosine term.
4. **(Repeated Pole)** \(\rightarrow\) Product of \(t\) and exp. term.
Find the TF to the electrical system.

\[ V_i \quad \frac{1}{\frac{1}{SC}} \quad V_o \]

**Solution:**

\[ \text{SL} || R = \frac{RSL}{R + SL} \]

\[ \frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{SC}}{\frac{1}{SC} + \frac{RSL}{R + SL}} \]

\[ \frac{V_o(s)}{V_i(s)} = \frac{R + SL}{R + SL + S^2RLC} \]

**Q-2**

\[ V_i \quad \frac{1}{\Omega} \quad 1H \quad \frac{1}{\Omega} \quad 1F \quad \frac{1}{\Omega} \quad V_o \quad \frac{1}{2F} \]

**Solution:**

\[ \frac{(1 + \frac{1}{5})}{\frac{1}{5}} \cdot \frac{(1 + S)}{1 + \frac{1}{5} + 1 + S} \]

\[ = \frac{(1 + S + \frac{1}{5} + S)}{(1 + \frac{1}{5} + 1 + S)} = 1. \]
\[ \frac{V_0(s)}{V_i(s)} = \frac{1 + \frac{1}{2s}}{1 + 1 + \frac{1}{2s}} = \frac{2s+1}{4s+1} \]

\[ V_0(s) = \frac{10^5}{s^2 + 10^5 s + 1} \]

**Solution:**
\[ \frac{V_0(s)}{V_i(s)} = \frac{1}{sLC + sCR + 1} \]
\[ = \frac{1}{s^2 \times 10^{-5} + s + 1} \]
\[ V_0(s) = \frac{10^5}{s^2 + 10^5 s + 1} \]

**Solution:**
By KVL,
\[ V_i(s) = z_1(s) \cdot I_1(s) + z_2(s) \cdot [I_1(s) - I_2(s)] \]
\[ V_i(s) = [z_1(s) + z_2(s)] I_1(s) - z_2(s) \cdot I_2(s) \cdot 0 \]

By KVL,
\[ z_3(s) \cdot I_2(s) + z_4(s) \cdot I_2(s) + [I_2(s) - I_1(s)] z_2(s) = 0. \]
\[ \therefore [z_3(s) + z_4(s)] z_2(s) I_2(s) = z_2(s) I_1(s) = 0. \]
\[ V_0(s) = I_2(s) \cdot Z_4(s). \]

\[
\begin{bmatrix} V_i \\ 0 \end{bmatrix} = \begin{bmatrix} z_1(s) + z_2(s) & -z_2(s) \\ -z_2(s) & z_2(s) + z_3(s) + z_4(s) \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}.
\]

\[ \rightarrow \text{By using Cramer's rule,} \]

\[ I_2 = \frac{\Delta_2}{\Delta} \]

\[
\begin{align*}
\Delta_2 &= \begin{vmatrix} z_1 + z_2 & V_i \\ -z_2 & 0 \end{vmatrix} \\
\Delta &= \begin{vmatrix} z_1 + z_2 & -z_2 \\ -z_2 & z_2 + z_3 + z_4 \end{vmatrix}
\end{align*}
\]

\[ I_2 = \frac{+V_i \cdot z_2}{(z_1 + z_2)(z_2 + z_3 + z_4) - z_2^2}. \]

\[ I_2 = \frac{V_1 \cdot z_2}{z_1 \cdot z_2 + z_1 \cdot z_3 + z_1 \cdot z_4 + z_2 \cdot z_3 + z_2 \cdot z_3} + z_2 \cdot z_4 - \frac{z_2^2}{z_1^2} \]

\[ I_2 = \frac{V_1 \cdot z_2}{z_1 \cdot (z_2 + z_3 + z_4) + z_2 \cdot (z_3 + z_4)} \]

\[ V_0(s) \quad \text{and} \quad V_1(s) \quad \text{are} \]

\[ V_0(s) = \frac{Z_2 \cdot Z_4}{Z_1 \cdot Z_1 \cdot (z_2 + z_3 + z_4) + z_2 \cdot (z_3 + z_4)} \]

\[ \text{H.B.} \]
Problem 1:

Find the TF.

\[ \frac{V_o(s)}{V_i(s)} = \frac{1}{s \cdot 1.4} \times \frac{1}{s \cdot 1.4} \]

\[ = \frac{1}{s^2} \left( \frac{1}{s \cdot 1.4} + 1 + \frac{1}{s \cdot 1.4} \right) + \frac{1}{s \cdot 1.4} \left[ 1 + \frac{1}{s \cdot 1.4} \right] \]

\[ = \frac{1}{s^2} \left( \frac{1}{s \cdot 1.4} \right) \left( s + 1 \right) + \left( \frac{1}{s} + \frac{1}{s^2} \right) \]

\[ = \frac{1}{(s + 1)(s + 1)} \]

\[ \frac{V_o(s)}{V_i(s)} = \frac{1}{s^2 + 3s + 1} \]

Problem 2:

Find the TF.

\[ \frac{V_o(s)}{V_i(s)} = \frac{V_o(s)}{V_o'(s)} \times \frac{V_o'(s)}{V_i'(s)} \]

\[ \Rightarrow \frac{V_o'}{V_i} = \frac{R}{R + \frac{1}{s c}} = \frac{R s c}{1 + R s c} = \frac{0.5 s}{1 + 0.5 s} = \frac{s}{s + 2} \]
\[ \frac{V_0(s)}{V_1(s)} = \frac{1}{1 + 5CR} = \frac{1}{1 + s/2} = \frac{2}{s + 2}. \]

\[ \frac{V_0(s)}{V_1(s)} = \frac{S}{(s + 2)} \times \frac{2}{(s + 2)^2} = \frac{2S}{s^2 + 4s + 4} \]

\[ \frac{V_0(s)}{V_1(s)} = \frac{2S}{(s + 2)^2} \]

\[ \text{TF of the OP-AMP:} \]

1. **Inverting OP-Amp:**

2. **Non-inverting OP-Amp:**
\[
\frac{V_o(s)}{V_i(s)} = 1 + \frac{Z_2(s)}{Z_1(s)}
\]

**Q.** Find the TF.

\[
\begin{align*}
\text{Som}:
Z_1(s) &= \left(\frac{1}{sC}\right) R(s) = \frac{1}{sC} \frac{R}{R + \frac{1}{sC}} = \frac{R}{1 + sCR} \\
\Rightarrow Z_1(s) &= \frac{1}{1 + s(1M \cdot 1M)} = \frac{1M}{s+1} \\
\Rightarrow Z_2(s) &= R + \frac{1}{sC} = 1M + \frac{1}{0.5MS} = 1M + \frac{2}{1\mu S} \\
&= \frac{s+2}{s1\mu} = 1M \left(\frac{s+2}{s}\right)
\end{align*}
\]

\[
\therefore \frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)} = -\frac{1M \left(\frac{s+2}{s}\right)}{1M/(s+1)}
\]

\[
\therefore \frac{V_o(s)}{V_i(s)} = -\frac{(s+1)(s+2)}{s}
\]

\[
\therefore \frac{V_o(s)}{V_i(s)} = -\frac{s^2 + 3s + 2}{s}
\]
Find the TF.

\[ Z_1(s) = R + sL = (1 + s). \]
\[ Z_2(s) = \frac{R}{1 + sC} = \frac{1}{1 + sC} \cdot \frac{R}{s} \]
\[ \frac{V_o(s)}{V_i(s)} = 1 + \frac{Z_2(s)}{Z_1(s)} = 1 + \frac{1}{(s+1)^2} \]
\[ \frac{V_o(s)}{V_i(s)} = \frac{s^2 + 2s + 2}{s^2 + 2s + 1} \]

* TF to the Differential Equations:

Write the TF to the given system.

Where \( x \) is input and \( y \) is output.

1. \[
\frac{d^3y}{dt^3} + 5 \frac{d^2y}{dt^2} + 7 \frac{dy}{dt} + 9y = 2 \frac{dx}{dt} + x(t)T.
\]

Solution:
Take L.T.

\[ s^3y(s) + 5s^2y(s) + 7sy(s) + 9y(s) = 2sX(s) + e^{-sT}X(s). \]
\[ (s^3 + 5s^2 + 7s + 9)y(s) = (2s + e^{-sT})X(s). \]
\[ TF = \frac{Y(s)}{X(s)} = \frac{2s + e^{-5s}}{s^3 + 5s^2 + 3s + 9} \]

\[ 2 \frac{d^3 y}{dt^3} + 2 \frac{d^2 y}{dt^2} + \gamma \frac{dy}{dt} + 10 = \frac{dx}{dt} + x. \]

**Solution:** The given system is **non-linear**, hence **TF is not defined.** [H.E.

**a.** Write the differential equation to the given TF.

\[ \frac{Y(s)}{X(s)} = \frac{2s + 2}{s^2 + 5s + 6} \]

**Solution:**

\[ Y(s) (s^2 + 5s + 6) = (2s + 3) X(s). \]

\[ \Rightarrow s^2 Y(s) + 5s Y(s) + 6Y(s) = 2s X(s) + 3X(s). \]

\[ \Rightarrow \frac{d^2 y}{dt^2} + 5 \frac{dy}{dt} + 6y = 2 \frac{dx}{dt} + 3x. \]

**TF to the Signal Response:**

To get the TF from the signal response used the following formula:

\[ TF = \frac{L[y(t)]}{L[x(t)]} \bigg|_{t=0}. \]
The unit step response of the system is

\[ y(t) = \left( \frac{5}{2} - \frac{5}{2} e^{-2t} + 5t \right), \quad t > 0 \]

its TF is _?_

**Solution:**

\[
\text{TF} = \frac{L \left[ \text{Unit Step Resp.} \right]}{L \left[ \text{Unit Step} \right]}
\]

\[
\therefore \text{TF} = \frac{5s}{2s} - \frac{5}{2(s+2)} + \frac{5}{s^2}
\]

\[
\frac{1}{5}
\]

\[
= \frac{5s(s+2) - 5s^2 + 5(s+2)}{2(s+2)}
\]

\[
\therefore \text{TF} = \frac{10s + 10}{2s(s+2)}
\]

\[
\therefore \text{TF} = \frac{5(s+1)}{5(s+2)}
\]
The impulse response of the system is:
\[ c(t) = (-4e^{-t} + 6e^{2t}) \] for \( t \geq 0 \). The equivalent step response is:

\[ \text{Step response is } \int_{0}^{t} (-4e^{-x} + 6e^{2x}) \, dx. \]

\[ = \left[ 4e^{-x} - 3e^{2x} \right]_{0}^{t}. \]

\[ = [4e^{-t} - 3e^{2t} - 4 + 3]. \]

\[ y(t) = 4e^{-t} - 3e^{2t} - 1. \]

\[ \star \text{Sensitivity:} \]

The sensitivity gives the relative variations in the output due to parameter variations in (i) \( \alpha \) and (ii) \( \beta \).

\[ \Rightarrow \text{Sensitivity of the TF w.r.t. } \alpha. \]

\[ G(s) \Rightarrow S_{\alpha} = \frac{\text{\% Change in TF}}{\text{\% Change in } \alpha}. \]

\[ S_{\alpha} = \frac{\Delta G}{\Delta \alpha} = \frac{G}{\alpha} \times \frac{\Delta G}{\Delta \alpha}. \]

Similarly,

\[ S_{\beta} = \frac{H}{T} \times \frac{\Delta H}{\Delta \beta}. \]
Find the sensitivity of the OL and CL sys w.r.t. Variations. (i) $C(s)$, (ii) $H(s)$.

**Solution:**

1. **OL Sys.**

   \[ S_{\alpha T} = \frac{C_r}{T} \times \frac{\Delta T}{\Delta \alpha} = \frac{C_r}{\alpha} \cdot \frac{\Delta \alpha}{\Delta \alpha} = 1. \]

2. **CL Sys.**

   \[ T = \frac{C_r}{1+\alpha H}. \]

   \[ S_{\alpha T} = \frac{C_r}{T} \times \frac{\Delta T}{\Delta \alpha} = \frac{C_r}{\alpha} \times \left(1 + \alpha H\right) \times \frac{(1+\alpha H)(1) - (\alpha H)(H)}{(1+\alpha H)^2} \]

   \[ S_{\alpha T} = \frac{1}{1+\alpha H} \] \[ \text{H.B.} \]

\[ S_{\alpha H} = \frac{H}{T} \times \frac{\Delta T}{\Delta \alpha} = \frac{H}{\alpha} \times \left(1 + \alpha H\right) \times \frac{-C_r}{H} \]

\[ S_{\alpha H} = \frac{-C_r H}{(1+\alpha H)(1+\alpha H)} \] \[ \text{H.B.} \]

\[ S_{\alpha H} > S_{\alpha T} \]

\[ \Rightarrow \text{feedback is more sensitive than the forward path.} \]
Find the sensitivity of the system w.r.t. variations in \( K \) and \( \alpha \).

\[
\frac{\Delta (CS)}{\Delta RCS} = \frac{K}{S(S\alpha + K) + K} = \frac{K}{S^2 + S\alpha + K}
\]

(i) \( S^K_T = \frac{K}{T} \times \frac{\partial T}{\partial K} \)

\[
S^K_T = \frac{K}{S(S^2 + S\alpha + K)} \times \frac{(S^2 + S\alpha + K) (1) - (K)(1)}{(S^2 + S\alpha + K)^2}
\]

\[
S^K_T = \frac{S^2 + \alpha S}{S^2 + \alpha S + K}
\]

(ii) \( S^\alpha_T = \frac{K\alpha}{T} \times \frac{\partial T}{\partial \alpha} \)

\[
S^\alpha_T = \frac{K\alpha}{K} \times \frac{(S^2 + S\alpha + K)}{(S^2 + S\alpha + K)^2} \times \frac{-K \times S}{(S^2 + S\alpha + K)^2}
\]

\[
S^\alpha_T = -\frac{\alpha S}{(S^2 + \alpha S + K)^2}
\]
Block Diagram:

- The purpose of the block diagram is to find the overall TF of the system.

- A block diagram is nothing but the shorthand pictorial representation of the system between input and output.

\[ R(s) \xrightarrow{C(s)} C(s) \]

- The systems can be represented in two ways:
  1. Open Loop form
  2. Closed Loop form

1. **Open Loop form:**

\[ R(s) \xrightarrow{C(s)} C(s) \]
2. **Closed Loop Form**: 

\[ \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} \]

**Feedback Signal**: 

\[ E(s) = R(s) - C(s) \]

**Forward Path Gain**: 

\[ \frac{G(s)}{E(s)} = \frac{C(s)}{R(s)} \]

**Feedback Path Gain**: 

\[ \frac{B(s)}{C(s)} = \frac{H(s)}{E(s)} \]

**Loop Gain (Open Loop Gain)**: 

\[ G(s)H(s) : \text{loop gain} \]

**Open Loop Transfer Function (OLTF)**: 

\[ (G(s)H(s)) \Rightarrow \text{OLTF of a Non-unity FFB System} \]

\[ H(s) = 1 \Rightarrow G(s) \Rightarrow \text{OLTF of a Unity FFB Sys.} \]

**The factor** 

\[ G(s)H(s) \text{ represent the actual closed loop system. It is also called as loop gain (open loop gain).} \]
In a practical system the phase shift between feedback signal and input signal is 0° (or) 360° whereas for -ve feedback the phase shift between input and feedback signal is ±180° (or) out of phase.

* Comparison between open loop system and closed loop system.

Open Loop System

- Gain: \[ G(s) \]
- \[ R(s) \rightarrow C(s) \]
- \[ C(s) = \frac{C(s)}{R(s)} \]

Closed Loop System

- The main disadvantage of 
  FIB is the gain is reduced by the factor
- \[ \frac{G(s)}{1 + G(s)H(s)} \]
- \[ \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} \]

* Stability:
- Stability is a notion that describes whether the system will be able to follow the input command.

Open Loop system is more stable.

→ The CL sys. stability depends on the loop gain.
* **Accuracy:**

⇒ The OL sys. accuracy depends on the IIP and process.
⇒ The OL sys. is less accurate.

* **Sensitivity:**

⇒ The OL system is highly sensitive w.r.t. the disturbance, noise and environmental conditions because whenever changes occur in the system it directly affects the IIP.
⇒ If $\alpha H = -1$ then the CL sys. stability is affected.
⇒ If $\alpha H = 0$ then CL sys. stability = OL sys. stability.
⇒ If $\alpha H > 0$ then the CL sys. more stable than the OL sys.
⇒ The CL sys. accuracy depends on the F1B new ratio.
⇒ If the F1B new gives the stable value then the CL sys. becomes highly more accurate than OL sys.

⇒ The closed loop sensitivity decreases by the factor of $1 - (\alpha S) H(S)$. i.e. the changes in IIP due to the disturbance, noise and environmental conditions is very less.
For any practical system the gain $BW$ product is constant.

$$BW \propto \frac{1}{C_s} \approx \frac{0.25}{C_s}$$

With feedback the gain is decreased by the factor of $1+CF$. That means the BW increases by $1+CF$.

The large BW gives the very quick response.

The CL sys. gives the very quick response compared to the OL sys.

*Reliability:

The reliability completely depends on the no. of discrete components used in the system.

The open loop sys. is more reliable as it has less no. of components.

In OL system it is not necessary to measure the output.

It is less reliable than OL system.

The output must be measured, error are generated, sensors are
Block Diagram Reduction Techniques:

1. **Blocks are in Series (or) Cascade:**

   \[ E_1 \rightarrow G_{r1} \rightarrow E_2 \rightarrow G_{r2} \rightarrow E_3 \rightarrow G_{r3} \rightarrow C \]

   \[ E_1 + E_2 + E_3 = G_{r1} \cdot G_{r2} \cdot G_{r3} \rightarrow C \]

   \[ \log \rightarrow x \leftrightarrow + \]

   \[ \div \leftrightarrow - \]

2. **Blocks are in Parallel:**

   \[ R \]

   \[ G_{r1} \]

   \[ E_1 \]

   \[ G_{r2} \]

   \[ E_2 \]

   \[ G_{r3} \]

   \[ E_3 \]

   \[ R \]

   \[ C \]

   \[ \frac{E_1, E_3}{E_2} \to G_{r1}, G_{r2}, G_{r3} \to C \]

3. **Loop:**

   \[ R(s) \]

   \[ X \]

   \[ G(s) \]

   \[ H(s) \]

   \[ R(s) \rightarrow C(s) \]

   \[ \frac{G(s)}{1 + G(s) \cdot H(s)} \to \]

   \[ C(s) \]
4. Interchanging Ob Summing Points:

\[ A \pm B \pm C = A \pm c \pm B. \]

5. Adjusting the Block Gain and Take Off Point:

(i)

\[ R \]
\[ c_1 \]
\[ c_2 \]
\[ c_3 \]
\[ c = c_1(c_2+c_3) \]
\[ T = c_1(c_2+c_3)R \]

(ii)

\[ R \]
\[ c_1 \]
\[ c_2 \]
\[ \frac{1}{c_3} \]
\[ C = \frac{c_1 c_2}{c_3} \]
(i) Adjusting Block Gain & Summing Point.

\[ R \rightarrow Cr \rightarrow Cr_2 \rightarrow \frac{1}{Cr_3} \rightarrow C \]

\[ T = R \]

\[ C = RCr \pm B \]

(ii) 

\[ R \rightarrow Cr \rightarrow \frac{R + \frac{B}{Cr}}{\frac{1}{Cr}} \rightarrow C \]

\[ C = RCr \pm B \]

\[ B \]

\[ C = Cr \pm B \]

\[ C = Cr \pm B \]
Diagram (i) Adjusting the Summing Point & Take Off Point.

Diagram (ii)

Steps:

1. \( S_1 \rightarrow \) Series || Hcl || Loop.
2. \( S_2 \rightarrow \) \( + \rightarrow X \rightarrow + \)
3. \( S_3 \rightarrow \) \( X \rightarrow C_{\alpha_1} \rightarrow C_{\alpha_2} \)
A) Find TF:

\[ \frac{C(s)}{R(s)} = \frac{\frac{C_2}{1 + \alpha_2 H_2}}{1 + \alpha_2 + \alpha_3} \]

H_1

\[ \frac{C_1 \cdot \alpha_2 \cdot (1 + \alpha_2 + \alpha_3)}{1 + \alpha_2 \cdot H_2} \]
\[ \Rightarrow \quad \frac{C(s)}{R(s)} = \frac{G_1 \cdot G_2}{1 + G_2 H_2 + H_1 \cdot G_1 \cdot G_2} \quad \frac{1 + G_3 + G_4}{1 + G_3 + G_4} \]

**Find** \( T_F \).

\[ \text{Solution:} \]

\[ \frac{C(s)}{R(s)} = \]

\[ 1 + C_1 H_1 - C_1 C_2 H_2 + C_1 C_2 C_3 H_3 = C_1 C_2 C_3 C_4 \]

\[ 1 \]

\[ \]
Draw the block diagram for the following.

**Solution:**

Note: While doing the shifting operation, the changes occur only in addition to the forward path and feedback path connected to that point only.

Before shifting and after shifting, forward path gain should remain same. We don't want to loose and we don't want to any...
extra gain. So, if it is extra gain after shifting then divide and if it is we loose any gain, we should multiply.

(a) Find the TF.

So m: We have 2 option:

(i) Shifting $T_1$ after $c_3$.

→ Before shifting there are three blocks $c_1, c_2, \text{ and } H_2$.

→ After shifting there are four blocks $c_1, c_2, c_3, \text{ and } H_2$. So, we should divide $c_3$ by $c_3$.

(ii) Shifting $T_2$ before $c_3$.

→ Before shifting there are four blocks $c_1, c_2, c_3, \text{ and } H_2$.

→ After shifting they becomes 3 blocks. i.e. $c_1, c_2, \text{ and } H_2$. So, we should multiply
(i) Method - 1:

\[ \frac{C(s)}{R(s)} = \frac{C_1 \cdot C_2 \cdot C_3 \cdot (C_3 + C_4) \cdot C_5}{1 + C_2 \cdot C_3 \cdot H_2 + C_5 \cdot H_1 + C_1 \cdot C_2 \cdot (C_3 + C_4) \cdot C_5} \]

(ii) Method - 2
\[
\frac{C(s)}{R(s)} = \frac{c_r_1 \cdot c_r_2 \cdot (c_r_3 + c_r_4) \cdot c_r_5}{1 + c_r_2 \cdot H_2 \cdot c_r_3 + c_r_5 \cdot H_1 + c_r_1 \cdot c_r_2 \cdot (c_r_3 + c_r_4) \cdot c_r_3}
\]

Find the TF.
\( C(s) \) = \frac{C_1 + C_3}{C_1}

\( p(s) \) = \frac{C_1 \cdot C_2}{1 + C_2 \cdot H_2 + C_1 \cdot C_2 \cdot H_1}

\( C(s) \) = \frac{C_2 \cdot (C_1 + C_3)}{1 + C_2 \cdot H_2 + C_1 \cdot C_2 \cdot H_1}

Find the o/p due to the multi i/p.

Sum: By superposition theorem it can be solved, i.e., take only one input at a time keeping all others zero.
(i) $R_1, R_2 = 0, R_3 = 0$

\[
\begin{align*}
C(s) &= \frac{cr_1 \cdot cr_2}{1 + cr_1 \cdot cr_2 \cdot H_1 \cdot H_2} \\
\end{align*}
\]

(ii) $R_2, R_1 = 0, R_3 = 0$

\[
\begin{align*}
\Rightarrow \quad \frac{C}{R_2} &= \frac{cr_2}{1 - (cr_1 \cdot H_1 \cdot cr_2 \cdot H_2)} \\
&= \frac{cr_2}{1 + cr_1 \cdot cr_2 \cdot H_1 \cdot H_2} \\
\end{align*}
\]

(iii) $R_3, R_1 = 0, R_3 = 0$

\[
\begin{align*}
\end{align*}
\]
\[ \frac{C(s)}{R(s)} = \frac{-H_1 \cdot c_1 \cdot c_2}{1 + c_1 \cdot c_2 \cdot H_1 \cdot H_2} \]

\[ C = \frac{R_1 \cdot c_1 \cdot c_2 + R_2 \cdot c_2 - R_3 \cdot c_1 \cdot c_2 \cdot H_1}{1 + c_1 \cdot c_2 \cdot H_1 \cdot H_2} \]

**Question:** Find the gain of the system given below:

\[ \frac{C(s)}{R(s)} = -40 + 2.95 \approx -37. \]
**Note:** In the above example, the oil at \( T_{P_1} \) is equal to the oil at \( T_{P_2} \) at any instant for any \( \Delta t \). So, they can be interchanged as follows.

\[
\frac{C(s)}{R(s)} = \frac{CR}{1 - CRH} + \frac{CR}{1 + CRH}
\]

\[
\frac{C(s)}{R(s)} = \frac{2CR}{1 - CRH}
\]
The impulse response of the unity feedback system is
\[ c(t) = (-t.e^{-t} + 2e^{-t}) \]. The open loop TF equal to?

**Sln:** Mention \( FIB \) is a CLOTF.

\[
\frac{C(s)}{R(s)} = \frac{-1}{(s+1)^2} + \frac{2}{s+1}
\]

\[ R(s) = \frac{1}{s} \quad (\therefore \text{Impulse}). \]

\[
\frac{C(s)}{R(s)} = -\frac{s-1 + 8s + 2s^2}{s^2 + 2s + 1}
\]

\[
\frac{C(s)}{1+Clh} = \frac{2s+1}{s^2 + 2s + 1}
\]

\[ \frac{C(s)}{s} = \frac{2s+1}{s^2} \leftarrow \text{OLTF}. \]

**Q2** Find the \( \text{OL DC gain} \) of a unity FIB system. at Closed loop TF.

\[
\frac{C(s)}{R(s)} = \frac{2s+4}{s^2 + 6s + 13}
\]

**Sln:**

\[ C(s) = \frac{2s+4}{s^2 + 4s + 8} \leftarrow \text{OLTF} \]

for \( \text{D.c.} \Rightarrow s=0. \)

\[ \Rightarrow \text{OL gain} = \frac{4}{1} = 4. \]
The impulse response of a system is $5e^{-2t}$. To produce the response as $te^{-2t}$, the Laplace must be equal to $-\frac{1}{5(s+2)}$.

So we have:

$$g(t) = 5e^{-2t},$$
$$c(t) = te^{-2t}.$$

Thus:

$$G(s) = \frac{c(s)}{R(s)}.$$

Thus:

$$R(s) = \frac{C(s)}{G(s)}.$$

Thus:

$$R(s) = \frac{1}{(s+2)^2} = \frac{1}{5(s+2)}.$$

Thus:

$$\hat{g}(t) = \frac{1}{5}e^{-2t}.$$

Thus:

$$\alpha(t) = 0.2e^{-2t}.$$
**Signal Flow Graph (SFG):**

*Purpose:*
- To bind the overall TR of the system.
- SFG is the graphical representation of the set of linear algebraic equations between input and output.
- The SFG analysis is developed to avoid the mathematical calculation like solving integer, differential equations (or) linear algebraic equations.

*Construction of SFG for the Linear Algebraic Equations:*

1. $y_2 = 10y_1$

\[
\begin{align*}
\text{node} &\quad y_2 = 10 \cdot y_1 \\
\text{10} &\quad \text{node} \\
\text{Gain} &\quad \text{Path gain (or) Transmittance.}
\end{align*}
\]
\( y_4 = 2y_1 + 5y_2 + 10y_3 \)

\( y_1 \quad \Rightarrow \quad y_2 \quad \Rightarrow \quad y_3 \quad \Rightarrow \quad y_4 \quad (\text{many to one}) \)

\( y_1 \quad \Rightarrow \quad y_2 \quad \Rightarrow \quad y_3 \quad \Rightarrow \quad y_4 \quad (\text{one to many}) \)

3. \( y_2 = 10y_1 \),
   \( y_3 = 20y_1 \),
   \( y_4 = 30y_1 \).

* Construct the SFA from the given sets of linear algebraic equations:

1. \( y_2 = y_1 + 2y_3 \),
   \( y_3 = 3y_1 + 4y_4 \),
   \( y_4 = 5y_2 + 6y_3 + 7y_5 \),
   \( y_5 = 8y_4 + 9y_2 \).
Q.2) Find the no. of forward paths, no. of individual loops, no. of two non-touching loops to the above signal graph.

```
Solution:
Forward path:
F₁ → 1 → 3 → 5 → 8.
F₂ → 1 → 3.
```

```
→ no. of individual loop:
L₁ : 3 → 2 (φ/3) L₄ : 6 (4)
L₂ : 5 → 4 (3/4) L₅ : 3 → 4 → 2 (φ/3, 4, 5).
L₃ : 8 → 7 (φ/5)
```

```
→ Two non-touching loop. (It is common node then touching otherwise non-touching).
O L₁ → L₂ X O L₂ → L₁ X
  L₃ L
  L₄ L
  L₅ X

O L₃ → L₁ L L₂ X
  L₄ X
  L₅ X

O L₄ → L₁ L L₂ X
  L₃ X
  L₅ X

O L₅ → L₁ L L₂ X
  L₃ X
  L₄ X
  L₅ X
```


Non-touching loop \( \Rightarrow \) 2.

\( L_1 L_2, L_1 L_4 \).

\[ \text{Loop:} \]

\[ \text{It is a path which terminate at the same node where it is started.} \]

Non-touching Loops:

\[ \text{If there is a no common node between two (or) more loops then it is said to be the non-touching loop.} \]

Forward path:

\[ \text{It is the path from input to output.} \]

Input node:

\[ \text{A node which has only outgoing branches is called input node.} \]

Output node:

\[ \text{A node which has only incoming branches is called output node.} \]

Chain (or) Link node:

\[ \text{The node which has both incoming} \]
and outgoing branch.

Note:

\[ \Rightarrow \text{The Condition to select the Correct Path (or Loop) is each node should be touch only once.} \]

\[ \Rightarrow \text{[Whenever many feedback are cascade with only one forward path it forms a Loop].} \]

\[ \text{Fig.}\]

\[ R(s) \rightarrow 1 \rightarrow 2 \rightarrow A \rightarrow 3 \rightarrow B \rightarrow 4 \rightarrow E \rightarrow 5 \rightarrow C \rightarrow G(s). \]

\[ \text{1.} \]

\[ D \Rightarrow \]

\[ 2 \text{ For} \]

\[ 3 \leftarrow 4 \rightarrow 2 \Rightarrow \text{(2L)} \]

\[ \Rightarrow L_1 = BD \rightarrow 3, 4 \]

\[ L_2 = (D) \rightarrow 3, 4. \]

\[ \text{2.} \]

\[ C \Rightarrow \]

\[ 3 \text{ For} \]

\[ 2 \leftarrow 5 \rightarrow 3 \Rightarrow (3L) \]

\[ \Rightarrow L_3 = ABE \rightarrow 2, 3, 4, 5 \]

\[ L_4 = A (EC) \rightarrow 2, 3, 4, 5 \]

\[ L_5 = fG \rightarrow 3, 5. \]

\[ \text{3 - NTL:} \]

\[ L_1 \rightarrow 5 (3, 4) \]

\[ L_2 \rightarrow 5 (3, 4) \]

\[ L_3 \rightarrow 5 (3, 4) \]

\[ L_4 \rightarrow 5 (3, 4) \]

\[ L_5 \rightarrow 5 (3, 4) \]
Som:  

\[ D \Rightarrow \overset{2 \text{ for }}{1} \quad \overset{3 \text{ for }}{A} \Rightarrow \overset{2L}{\rightarrow} \]

\[ L_2 = CD \Rightarrow 2, 3, 4. \]

\[ \rightarrow L_3 = B F G \Rightarrow 1, 2, 3, 4. \]

\[ \rightarrow L_4 = C (E G) \Rightarrow 1, 2, 3, 4. \]

\[ \rightarrow L_5 = F (G \Rightarrow 1, 4. \]

\[ 2 \text{ NTL} \]

\[ \begin{align*}
&L_6 L_1 L_5 = B D F G \Rightarrow 1, 2, 3, 4 \\
&L_6 L_2 L_7 = B D I \Rightarrow 2, 3, 4 \\
&L_2 L_5 = C D F G \Rightarrow 1, 2, 3, 4 \\
&L_2 L_6 = C O H \Rightarrow 1, 2, 3, 4 \\
&L_6 L_7 = C O I \Rightarrow 2, 3, 4, 4 \\
&L_1 L_6 = B O H \Rightarrow 1, 2, 3, 4 \\
&L_2 L_6 \Rightarrow A B E G \Rightarrow 1, 2, 3
\end{align*} \]
\[ H_1 \Rightarrow \quad \text{1 for} \quad \Rightarrow \quad \begin{array}{c} 4 \leftarrow \quad 5 \\ \text{2 for} \quad \end{array} \Rightarrow \quad \begin{array}{c} \text{1L} \Rightarrow \quad L_1 = -c_3 H_1 \Rightarrow 4_{15} \end{array} \]

\[ H_2 \Rightarrow \quad \begin{array}{c} 6 \leftarrow \quad 8 \quad \text{2 for} \end{array} \Rightarrow \quad \begin{array}{c} \text{2L} \Rightarrow \quad L_2 = -c_5 c_6 H_2 \Rightarrow 6_{13} 8_{18} \quad \text{L}_2 = -c_8 H_2 \Rightarrow 6_{18} \end{array} \]

\[ H_3 \Rightarrow \quad \begin{array}{c} 3 \leftarrow \quad 7 \quad \text{1 for} \end{array} \Rightarrow \quad \begin{array}{c} \text{2L} \Rightarrow \quad L_4 = -c_2 c_3 c_4 c_5 H_3 \Rightarrow 3_{18} 5_{16} 3_{18} \quad \text{L}_5 = -c_2 H_3 \Rightarrow 3_{18} \end{array} \]

\[ H_4 \Rightarrow \quad \begin{array}{c} 2 \leftarrow \quad 6 \quad \text{3 for} \end{array} \Rightarrow \quad \begin{array}{c} \text{2L} \Rightarrow \quad L_6 = -c_1 c_2 c_3 c_4 c_5 c_6 H_4 \Rightarrow 2_{13} 4_{15} 6_{17} 18 \end{array} \]

\[ 2 \text{NTL} \]

\[ L_1 L_2 \Rightarrow 4_{15} 6_{14} 18 \]
\[ L_1 L_3 \Rightarrow 4_{15} 6_{14} 18 \]
\[ L_1 L_5 \Rightarrow 3_{18} 4_{15} 7 \]
\[ L_1 L_7 \Rightarrow 2_{13} 4_{15} 6_{17} 18 \]
\[ L_3 L_5 \Rightarrow 3_{18} 4_{15} 6_{17} 18 \]

\[ 3 \text{NTL} \]

\[ L_1 L_2 L_5 \Rightarrow 3_{18} 4_{15} 6_{17} 18 \]
Mason's Gain Formula:

**Purpose:**
(i) To find the overall TF of the system.
(ii) To find the ratio of any two nodes.

\[
\text{Overall TF} = \sum_{k=1}^{i} \left( \frac{P_k \cdot \Delta_k}{\Delta} \right).
\]

Where, \( P_k \): \( k \)th forward path gain.

\[ \Delta = 1 - \sum (\text{individual loop gain}) \]

\[ + \sum (\text{sum of gain product of two non-touching loop}). \]

\[ - \sum (\text{sum of gain product of three non-touching loop}). \]

\[ + \sum (\text{sum of gain product of four non-touching loop}). \]

\[ \Delta_k = \Delta_k \text{ is obtained by removing the loops touching the } k\text{th forward path}. \]
Find the TF to the given signal flow graph.

Solution:

\[ P_1 = \alpha_1, \alpha_2, \alpha_3, \alpha_4 \]

\[ P_2 = \alpha_5 \]

**Loops:**

\[ L_1 = -H_1 \]
\[ L_2 = -\alpha_5 H_2 \]
\[ L_3 = -\alpha_4 H_3 \]

**2 NTL:**

\[ L_1 L_2 = \alpha_3 H_1 H_2 \]
\[ L_1 L_3 = \alpha_4 H_1 H_3 \]

\[ \Delta = 1 - (L_1 + L_2 + L_3) + (L_1 L_2 + L_1 L_3) \]

\[ \Delta = 1 + H_1 + \alpha_3 H_2 + \alpha_4 H_3 + \alpha_3 H_1 H_2 + \alpha_4 H_1 H_3 \]

\[ \Rightarrow \Delta_1 = 1 \]

\[ \Delta_2 = 1 - (L_1 + L_2) + (L_1 L_2) \]

\[ \Delta_2 = 1 + H_1 + \alpha_3 H_2 + \alpha_3 H_1 H_2 \]

**TF:**

\[ \boxed{\frac{\alpha_1 \alpha_2 \alpha_5 \alpha_4}{1+H_1 + \alpha_3 H_2 + \alpha_4 H_3 + \alpha_3 H_1 H_2 + \alpha_4 H_1 H_3} + \alpha_5 (1+H_1 + \alpha_3 H_2 + \alpha_3 H_1 H_2)} \]
Note:

→ In \( \Delta (y) \Delta k \), take the opposite sign for odd no. of non-touching loops and take the same sign for even no. of non-touching loops.

-2 Find the TF.

\[ TF = \frac{C_{r_1} \cdot C_{r_2} \cdot C_{r_3} \cdot C_{r_4} \cdot (1)}{1 + C_{r_1} \cdot H_1 + C_{r_3} \cdot H_3 + C_{r_2} \cdot C_{r_3} \cdot H_2} \]
\[
TF = \frac{c_{r_1} \cdot c_{r_2} \cdot c_{r_3} \cdot c_{r_4} + c_{r_5} \cdot c_{r_5}}{1 + c_{r_1} \cdot c_{r_2} \cdot H_1 + c_{r_2} \cdot c_{r_3} \cdot H_2 + c_{r_4} \cdot H_3 + c_{r_5} \cdot H_2 \cdot H_3 + c_{r_5} \cdot c_{r_4} \cdot H_1 \cdot H_3}.
\]
\[
\frac{C(s)}{R(s)} = \frac{(2 \cdot 3 \cdot 4) (1 + 0.5) + (5 \cdot 10) (1 + 3)}{1 + 3 + 4 + 24 + 50 + 0.5 \times (3/2 + 2) + (50 \cdot 3) + (0.5 \times 24)}
\]

\[
\frac{C(s)}{R(s)} = \frac{236}{248}
\]

Find the TF to the given Block Diagram by using Mason's gain formula.

\[
\frac{C(s)}{R(s)} = \frac{C_1 \cdot C_2 \cdot C_4 + C_1 \cdot C_2 \cdot C_3 + C_1 \cdot C_2 \cdot 1}{1 + C_2 \cdot H_1 + C_1 \cdot C_2 \cdot C_3 \cdot H_2 + C_1 \cdot C_2 \cdot C_3 \cdot H_2 + C_1 \cdot C}_2 \cdot H_2}
\]
\[
\frac{C(c_s)}{R(c_s)} = \frac{C_r_1 \cdot C_r_2 \cdot C_r_3 + C_r_4 (1 + C_r_2 \cdot C_H_2)}{1 + C_r_2 \cdot C_H_2 + C_r_1 \cdot C_r_2 \cdot C_r_3 \cdot C_H_1 + C_r_4 \cdot C_H_1 + C_r_4 \cdot C_H_1 \cdot C_r_2 \cdot C_H_2}
\]

\[
\frac{C(c_s)}{R(c_s)} = \frac{C_r_1 \cdot C_r_2 \cdot C_r_3 + C_r_4 \cdot C_r_2 \cdot C_r_3 + C_r_4 \cdot C_r_5}{1 + C_r_3 \cdot C_H_1 + C_r_2 \cdot C_r_3 \cdot C_H_1 \cdot C_H_2 + C_r_1 \cdot C_r_2 \cdot C_r_3 \cdot C_H_1 \cdot C_H_2 \cdot C_H_3}
\]
\[
\frac{C(s)}{R(s)} = \frac{c_1 c_2}{1 + c_3 H}
\]

\[
\frac{C(s)}{R(s)} = \frac{\alpha}{1 + c_1 H_1 + c_2 H + H_2}
\]

\[
\frac{C(s)}{R(s)} = \frac{\alpha + 1}{1 + \alpha H}
\]
\[
\frac{C(s)}{R(s)} = \frac{1}{s+2} = \frac{1}{s+2}
\]

\[
\frac{C(s)}{R(s)} = \frac{(2 \cdot 3 \cdot 4) + (5)(1 + 3)}{1 + 2 + 3 + 4 + 8 + 5}
\]

\[
\frac{C(s)}{R(s)} = \frac{4 \cdot 4}{2 \cdot 3}
\]
\[ \frac{C(s)}{R(s)} = \frac{1.1 \left( 1 + \frac{3}{s} + \frac{24}{s^2} \right)}{1 + \frac{2}{s} + 3/s + \frac{24}{s} + 6/s^2} \]
\[ \frac{C(s)}{R(s)} = \frac{s(s+27)}{s^2 + 29s + 6} \]

**Q:** Find \( \frac{y_2}{y_1} \), \( \frac{y_3}{y_1} \), \( \frac{y_5}{y_1} \), \( \frac{y_7}{y_1} \), \( \frac{y_8}{y_3} \), \( \frac{y_5}{y_2} \), and so on until any two nodes.

and so on, etc.

**Soln:**

\[ \text{(i) } \frac{y_6}{y_1} \]

\[ \frac{y_6}{y_1} = \frac{c_1 \cdot c_2 \cdot c_3 \cdot c_4 + c_1 \cdot c_5 \left( 1 + c_5 \cdot H_2 \right)}{(1 + c_1 \cdot H_1 + c_3 \cdot H_2 + H_4 + c_1 \cdot c_2 \cdot c_3 \cdot H_3 + c_1 \cdot H_1 \cdot c_3 \cdot H_2 + c_1 \cdot H_1 \cdot H_4 + c_3 \cdot H_2 \cdot H_4 + c_1 \cdot H_1 \cdot c_3 \cdot H_2 \cdot H_4) } \]
(iii) \( \frac{y_7}{y_1} = \frac{y_7}{\Delta} = \frac{\frac{g_6}{y_1}}{\Delta} = \frac{C_1 \cdot C_2 \cdot C_3 \cdot C_4 + C_1 \cdot C_5 \cdot (1 + C_5 H_2)}{\Delta} \)

(iii) \( \frac{y_5}{y_1} = \frac{C_1 \cdot C_2 \cdot C_3 \cdot (1 + H_4)}{\Delta} \)

(iv) \( \frac{y_2}{y_1} = \frac{1 \cdot (1 + C_3 H_2 + H_4 + C_3 H_2 \cdot H_4)}{\Delta} \)

(v) \( \frac{y_7}{y_2} \)

\[ \Rightarrow \text{NOTE: } \text{The Mason's gain formula gives the ratio w.r.t. input only. It can not give the middle nodes directly w.r.t. ratio.} \]

\[ \Rightarrow \frac{y_7}{y_2} = \frac{y_7 + y_1}{y_2 + y_1} \]

\[ \Rightarrow \frac{y_7}{y_2} = \frac{C_1 \cdot C_2 \cdot C_3 \cdot C_4 + C_1 \cdot C_5 \cdot (1 + C_5 H_4)}{(1 + C_3 H_2 + H_4 + C_3 H_2 \cdot H_4)} \]
\[ \frac{y_5}{y_2} = \frac{y_5/y_1}{y_3/y_1} \]

\[ \frac{y_5}{y_3} = \frac{c_1 c_2 c_3 c_4 (1 + H_4)}{c_1 c_4 + c_3 H_2 + c_3 c_4 H_4} \]

\[ \frac{y_5}{y_4} = \frac{y_5/y_1}{y_4/y_1} \]

\[ \frac{y_5}{y_5} = \frac{c_1 c_2 c_3 c_4 (1 + H_4)}{c_1 c_2 c_4 (1 + H_4)} \]

\[ \rightarrow \frac{y_5}{y_5} = c_{r_3}. \]

**Note:** In the above signal flow graph, node \( R \) is not an input node. In this case, we require to assume a dummy input node with path gain of 1 as shown in fig.
\[ \frac{C}{R} = \frac{c_1 u}{R_{10}} = \frac{C_f}{1 + H_2} \]

**M-II:**

Old node equation:

\[ C = R_c - C H_2 \]

\[ C (1 + H_2) = R_c \]

\[ \frac{C}{R} = \frac{C_f}{1 + H_2} \]

**Q:** Find \( c_1/R_1, c_1/R_2, c_2/R_1, c_2/R_2 \) for the given multi-input multi-output system.

\[ \frac{C_1}{R_1} = \frac{c_1 \cdot c_2 (1 + c_3 c_4 \cdot H_3) + c_5 \cdot c_6}{1 + c_1 \cdot H_1 + c_2 \cdot H_2 + c_3 \cdot H_1 \cdot H_3 - c_5 \cdot c_6 \cdot H_1 \cdot H_2 + c_1 \cdot H_1 \cdot c_3 \cdot c_4 \cdot H_3 + c_2 \cdot H_2 \cdot c_3 \cdot c_4 \cdot H_3} \]
\[ \frac{C_1}{R_2} = \frac{C_{r_3} \cdot C_{r_6} \left( 1 + C_{r_1} \cdot R_1 \right)}{\Delta} \]

\[ \frac{C_2}{R_1} = \frac{C_{r_5} \cdot C_{r_4} \left( 1 + C_{r_2} \cdot R_2 \right)}{\Delta} \]

\[ \frac{C_2}{R_2} = \frac{C_{r_3} \cdot C_{r_4} \left( 1 + C_{r_1} \cdot R_1 + C_{r_2} \cdot R_2 \right)}{\Delta} \]

**Construction of SFCR to Electrical**

Now:

**Question:** Draw the SFCR.

![Circuit Diagram]

**Solution:**

- Select the branch current and node voltages.
- Apply Laplace transform to the new variables and elements.
- Write the eqns for unknown currents and unknown voltages.

\[ I_1(s) = \frac{V_i(s) - V_1(s)}{R_1} \]
\[ V_1(s) = I_2(s) \cdot \frac{1}{sC}. \]

But \[ I_2(s) = I_1(s) - I_3(s). \]

\[ \Rightarrow V_1(s) = \frac{1}{sC} \left( I_1(s) - I_3(s) \right) \quad \text{(2)} \]

\[ \Rightarrow I_3(s) = \frac{V_1(s) - V_0(s)}{sL}. \quad \text{(3)} \]

\[ \Rightarrow V_0(s) = R_2 \cdot I_3(s). \quad \text{(4)} \]

\[ TF_{E-NW} = TF_{B-D \,(ca)} \cdot TF_{sfa}. \]

* Procedure to draw SFG directly.*

→ The nodes in a SFG are nothing but the variable along the series path. (Branch).

→ Each Elements in electrical Nw give the 1 forward path and I ve feedback path, except the last element. The last element is giving the only
forward path.

→ Take inverse of the impedance to the series branch elements as a path gain and take the same impedance for shunt branch elements as a path gain.

Flow path

- Ve F1B path.

\[ V_i \rightarrow I_1 \xrightarrow{R_1} V_1 \xrightarrow{\frac{1}{R_2}} I_3 \xrightarrow{L_1} V_2 \xrightarrow{\frac{1}{L_2}} I_5 \xrightarrow{C_2} V_0 \]
\[ V_0 = \frac{1}{sc} \cdot I. \]

\[ I = I_1 + I_2 \]

\[ I = \frac{V_i - V_0}{R} + \frac{V_i - V_0}{sL} \]

\[ \frac{1}{sc} \]

\[ \frac{1}{R} \]

\[ \frac{1}{sc} \]

\[ \frac{-1}{sc} \]

\[ \frac{-1}{R} \]
Time Domain Analysis:

Purpose: To evaluate the performance of the system w.r.t. to the time.

* Time - Response:

- If the response of the system varies with respect to the time then it is called as time response.
- The time response is nothing but the sum of transient response and steady state response.

\[ TR(c(t)) = C_{tr}(t) + C_{ss}(t) \]

⇒ Find the transient and steady state forms in the given time response.
\[ C(t) = 10 + 2 \sin 2t + 3 \cos 3t + 4t \cdot e^{-4t} + 5.5 \cdot e^{-5t} \sin t + 6t \cdot e^{-6t} \cos t \]

\( \Rightarrow \) **Transient term:**

\( \Rightarrow \) It is part of the system that becomes 0 as \( t \) becomes very large.

\[ \lim_{t \to \infty} C(t) = 0 \]

\( \Rightarrow \) The term which consist exponentially decay always gives the transient terms.

\( \Rightarrow \) The poles which lies left hand side of the s-plane gives the transient terms.
⇒ **Steady State Response:**  
⇒ It is the part of the response that remains after the transient becomes the zero.

⇒ The pole which lies on the imaginary axis gives the steady state term.

⇒ $\frac{1}{s} \Rightarrow u(t)$

⇒ $\frac{1}{s^2 + 1} \Rightarrow \sin t$

⇒ **Time Response to the First Order System.**

⇒ $R(s) + \frac{1}{1/s} \Rightarrow C(s)$
\[ C(s) = \frac{1}{\tau s} \quad H(s) = 1. \]

\[ \therefore \text{Type-1 \& order-1.} \]

\[ \Rightarrow \text{Practical \( C_k \) for the first order system is \( R_C \), \( R_L \) \( C_k \) of LPF.} \]

\[ \Rightarrow \text{Impulse response:} \]

\[ a(t) = a(t) \]

\[ \Rightarrow R(s) = 1. \]

\[ \Rightarrow \frac{C(s)}{R(s)} = \frac{1}{\tau s + 1}. \]

\[ \Rightarrow C(s) = \frac{1}{\tau s + 1} \]

\[ = \frac{1}{\tau} \cdot \frac{1}{s + \frac{1}{\tau}} \]

\[ \Rightarrow C(t) = \frac{1}{\tau} \cdot e^{-\frac{t}{\tau}} \quad \text{Transient term.} \]

\[ \Rightarrow C(t) \]

\[ \frac{1}{\tau} \quad \text{exp decay} \]

\[ \text{The impulse response consist the transient term. Transient term consist the} \]
the system parameters.

→ Hence, the impulse response is called system response (or) natural response (or) forced free forced response.

*  

→ error is nothing but the deviation of the output from the input.

i.e. \( e(t) = x(t) - c(t) \).

*  

Steady state error \((e_{ss})\):

→ The error at \( t \rightarrow \infty \).

\[
e_{ss} = \lim_{t \rightarrow \infty} e(t).
\]

→ \( e_{ss} = \lim_{t \rightarrow \infty} x(t) - c(t) \).

\[
e_{ss} = \lim_{t \rightarrow \infty} s(t) - \frac{1}{\tau} e^{-t/\tau}.
\]

→ The impulse response not consist the any steady state terms. Hence we can not defined the steady state errors.

(Or) The impulse input not exist at \( t \rightarrow \infty \). Hence we can not compare the old with
\[ t \to \infty \]

Thus, the unit response is:

\[
R(s) = \frac{1}{s}.
\]

\[
\frac{C(s)}{R(s)} = \frac{1}{(1 + s\gamma)}.
\]

\[
C(s) = \frac{1}{s(1 + s\gamma)}.
\]

\[
\frac{1}{s} = A + \frac{B}{(1 + s\gamma)}.
\]

\[
1 = A(1 + s\gamma) + B.
\]

\[ s \to 0, \quad A = 1 \]
\[ s \to -\frac{1}{\gamma}, \quad B = -1 \gamma. \]

\[
C(s) = \frac{1}{s} - \frac{\gamma}{1 + s\gamma} = \frac{1}{s} - \frac{1}{s + \frac{1}{\gamma}}.
\]

\[
C(t) = \left(1 - e^{-\frac{t}{\gamma}}\right)u(t).
\]

\[ \text{In the response, the steady state term is because of the input in the response and the transient term because of the system.} \]
\[ e_{ss} = \lim_{t \to \infty} \sigma(t) - c(t) \]
\[ = \lim_{t \to \infty} 1 - 1 + e^{-\tau t} \]
\[ = 0 \]

Unit Ramp Response:
\[ \sigma(t) = t \Rightarrow R(s) = \frac{1}{s^2} \]
\[ \therefore \frac{C(s)}{s^2} = \frac{1}{s} \cdot \frac{1}{s(s + 1)} \]
\[ \Rightarrow \frac{1}{s^2 (s + 1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s + 1} \]
\[ \Rightarrow 1 = A(s + 1) + B + C s^2 \]
\[ \Rightarrow s \to 0, \quad s \to -\frac{1}{\tau} \]
\[ 1 = B \]
\[ 1 = \frac{C}{s^2} \Rightarrow C = \tau^2 \]
\[ \therefore \quad 1 = A(\tau + 1) + \tau + \tau^2 \]
\[ A(\tau, \tau) = -\tau (1 + \tau) \]

\[ \boxed{A = -\tau} \]

\[ C(s) = \frac{-\tau}{s} + \frac{1}{s^2} + \frac{\tau^2}{s\tau + 1} \]

\[ C(t) = t - \tau + \tau e^{-t/\tau} \]

\[ e_{SS} = \lim_{t \to \infty} \sigma(t) - C(t) \]

\[ = \lim_{t \to \infty} \tau - e^{-t/\tau} = \tau \]

\[ \boxed{e_{SS} = \tau} \]

\[ \Rightarrow \text{Unit Parabolic Response:} \]

\[ \Rightarrow \sigma(t) = \frac{t^2}{2} \cdot u(t) \]

\[ \Rightarrow R(s) = \frac{1}{s^3} \]

\[ \Rightarrow C(s) = \frac{1}{s^3} \cdot \frac{1}{s\tau + 1} \]
\[ \frac{1}{s^3(s \gamma + 1)} = \frac{A}{s^3} + \frac{B}{s^2} + \frac{c}{s} + \frac{d}{s \gamma + 1} \]

\[ \therefore 1 = A(s \gamma + 1) + Bs(s \gamma + 1) + cs^2(s \gamma + 1) + ds^3. \]

\[ \Rightarrow s \to 0 \quad \Rightarrow s \to -\frac{1}{\gamma}. \]

\[ \Rightarrow 1 = A \quad \Rightarrow 1 = -\frac{d}{\gamma^3} \Rightarrow d = -\gamma^3 \]

\[ \Rightarrow \text{co-efficient of } x \text{ given } \gamma. \]

\[ A \gamma + B = 0. \quad \Rightarrow \quad B = -\gamma \]

\[ \Rightarrow \text{co-efficient of } x^2 \text{ given } s^2. \]

\[ B \gamma + C \gamma = 0. \]

\[ C = -B \gamma \]

\[ C = \gamma^3 \]

\[ C(s) = \frac{1}{s^3} - \frac{\gamma}{s^2} + \frac{\gamma^2}{s} - \frac{\gamma^3}{s \gamma + 1} \]

\[ \therefore C(t) = \frac{t^2}{2} - t \gamma + \gamma^2 - \gamma^3 e^{-t/\gamma} \]

\[ e \gamma = \lim_{t \to \infty} \gamma(t) - C(t) \]

\[ = \lim_{t \to \infty} t \gamma - t^2 + e^{-t/\gamma} \]

\[ = \lim_{t \to \infty} t \gamma - t + e \]

\[ 

\[ e \gamma = \infty. \]
\* Sinusoidal Response:

\( e(t) = \frac{1}{2} u(t) \)  

\( \sigma(t) = \frac{1}{2} u(t) \)

\( c(t) \)

\( e_s \rightarrow \infty \)

\( t \)

\( C(t) \)

\( C(t) \)

\( \sigma(t) = A \sin(\omega t + \phi) \rightarrow C(t) = A \sin(\omega t + \phi + \theta) \)

\( \sigma(t) = A \cos(\omega t + \phi) \rightarrow C(t) = A \cos(\omega t + \phi + \theta) \)

\[ \boxed{\text{The CLTF of a LTI system}} \]

\[ \frac{C(s)}{R(s)} = \frac{1}{s+1} \]  

for \( \text{IIP} \) \( \sigma(t) = \sin(t) \) the steady state OIP is...
\[ f(t) = \sin(t). \]
\[ \Rightarrow \quad \omega = 1 \quad \text{rad/sec}. \]

\[
\frac{C(s)}{R(s)} = H(s) = \frac{1}{s+1}.
\]

\[ \Rightarrow \quad H(j\omega) = \frac{1}{1+j\omega} = \frac{1}{1+j} = \frac{1}{\sqrt{2}} \angle -45^\circ
\]

\[ \therefore \quad C(t) = \frac{1}{\sqrt{2}} \cdot \sin(t - 45^\circ). \]

\[ \frac{C(s)}{R(s)} = \frac{s+1}{s+2}, \quad \sigma(t) = 10 \cos(2t + 45^\circ). \]

Find \( C(t) = ? \).

\[ \sigma(t) = 10 \cos(2t + 45^\circ) \]
\[ \Rightarrow \quad \omega = 2 \quad \text{rad/sec}. \]

\[ \therefore \quad H(s) = \frac{s+1}{s+2} = \frac{1+j\omega}{2+j\omega} = \frac{1+2j}{2+2j} \]

\[ \therefore \quad H(j\omega) = \frac{\sqrt{5} \tan^{-1}(2)}{\sqrt{8} \tan^{-1}(1)} = \sqrt{\frac{5}{8}} \angle \left( \tan^{-1}(2) - 45^\circ \right) \]

\[ \therefore \quad H(j\omega) = \sqrt{\frac{5}{8}} \angle 18.43^\circ \]

\[ \therefore \quad C(t) = 10 \times \sqrt{\frac{5}{8}} \cdot (2t + 45^\circ + 18.43^\circ) \]

\[ \therefore \quad C(t) = 8 \cos(2t + 63.43^\circ). \]
A system \( \frac{Y(s)}{X(s)} = \frac{S}{S+p} \) as \( \text{an olp} \).

\[ Y(t) = 1 \cdot \cos \left( 2t - \frac{\pi}{3} \right) \]

\[ x(t) = p \cdot \cos \left( 2t - \frac{\pi}{2} \right) \]

then the system parameter \( p \) is.

\[ \text{So,} \quad \omega = 2 \text{ rad/sec.} \]

\[ H(s) = \frac{j\omega}{j\omega + p} = \frac{j2}{p + 2j} = \frac{2}{\sqrt{p^2 + 4}} (\cos \left( \frac{-\pi}{2} - \tan^{-1} \left( \frac{2}{p} \right) \right)) \]

\[ h(j\omega) = \frac{2}{\sqrt{p^2 + 4}} \times \angle \frac{2}{p} \cdot \cos \left( \frac{-\pi}{2} - \tan^{-1} \left( \frac{2}{p} \right) \right) \]

Now, \( Y(t) = x(t) \cdot h(t) \).

\[ 1 \cdot \cos \left( 2t - \frac{\pi}{3} \right) = \frac{2 \cdot p}{\sqrt{p^2 + 4}} \cdot \cos \left( \frac{\pi}{2} + 2t - \frac{\pi}{2} - \tan^{-1} \left( \frac{2}{p} \right) \right) \]

compute, mag. and phase,

\[ \frac{2 \cdot p}{\sqrt{p^2 + 4}} = 1 \]

\[ 2p = \sqrt{4 + p^2} \]

\[ 4p^2 = 4 + p^2 \]

\[ 3p^2 = 4 \]

\[ p^2 = \frac{4}{3} \]

\[ p = \pm \frac{2}{\sqrt{3}} \]

\[ p = \frac{2}{\sqrt{3}} \]
Time Response to the Second Order System:

\[ \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \]

Type-1, order - 2.

The Practical Ckt to the Second Order System is R-L-C Ckt of L.P.F.

\[ \frac{V_o(s)}{V_i(s)} = \frac{1}{sC} \cdot \frac{1}{R + sL + \frac{1}{sC}} \]

\[ \frac{V_o(s)}{V_i(s)} = \frac{1}{s^2LC + sCR + 1} \]
\[
\frac{V_0(s)}{V_i(s)} = \frac{1}{s^2 + s \frac{R}{L} + \frac{1}{LC}}
\]

\[\Rightarrow \quad \omega_n = \sqrt{\frac{1}{NLc}}\]

\[\Rightarrow \quad \omega_n = \frac{1}{NLC} \]

\[2\pi \omega_n = \frac{R}{L}\]

\[\Rightarrow \quad 2\pi = \frac{R}{L} \times \sqrt{\frac{c}{NL}}\]

\[\therefore \quad 2\pi = \frac{R}{2} \times \sqrt{\frac{c}{NL}}\]

\[\Rightarrow \quad \text{\(\omega_n\): is called Natural freq. or oscillation (or) Sustained oscillation (or) Undamped oscillation.}\]

\[\Rightarrow \quad D = \frac{1}{2\pi} = \frac{1}{R} \times \sqrt{\frac{c}{NL}} = \text{damping ratio.}\]

\[\Rightarrow \quad \text{It gives the ratio of energy lost to energy stored.}\]

\[\Rightarrow \quad 2\pi \omega_n = \text{damping factor (or) actual damping.}\]

2) The second order stable for \(\delta > 0\).

\[\Rightarrow \text{The second order system response completely depends on } \delta.\]
The second order system is stable for all the positive values of \( \delta > 0 \), because the poles lie in the L-H plane.

\[ \text{Impulse Response:} \]

\[ x(t) = \delta(t). \]

\[ \Rightarrow R(s) = 1. \]

\[ \therefore \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\delta\omega_n + \omega_n^2}. \]

\[ \therefore C(s) = \frac{\omega_n^2}{s^2 + 2\delta\omega_n + \omega_n^2}. \]

Case 1: \( \delta = 0 \Rightarrow \text{Undamped.} \)

\[ \therefore C(s) = \frac{\omega_n^2}{s^2 + \omega_n^2}. \]
3) By real part:
\[ \tau = \frac{1}{\omega_n} = \infty. \]

4) Non-repeated poles on the j\omega axis hence the system is Marginally Stable.

By ILT:
\[ e(t) = \omega_n \sin\omega_n t. \]

Constant Amp. and fre. of oscillation.
undamped osc. | Natural osc. | Sustained osc. \[ \omega_n \]
When $\xi = 0$, the second order system response is constant amplitude and free oscillation which are called undamped oscillation.

Any system which produced the undamped oscillation is called undamped system and the system becomes marginal stable.

The second order system nature completely depends on $\xi$. For example, if $\xi = 0$ the second order system nature is constant amplitude and freq. of oscillation around the input which never be changed by changing the input signal hence when $\xi > 0$ the second order system is called undamped system irrespective of all the inputs.

Similarly when $\xi > 0$ & $\xi < 1$ the system is called underdamped system.
\( \Rightarrow \xi = 1 \rightarrow \text{Critical damped system.} \)

\( \Rightarrow \xi > 1 \rightarrow \text{Overdamped system.} \)

\( \Rightarrow \xi = 0. \)

(i) \text{Impulse}

\[ c(t) \]

\( \Rightarrow \text{Unit step.} \)

\[ c(t) \]

\[ 1 \]

\( 0 \rightarrow t \)

(iii) \text{Unit Ramp:}

\[ c(t) \]

\( \Rightarrow \text{Unit parabolic:} \)

\[ c(t) \]

\( t \rightarrow t \)
When \( s = 0 \) we can not find the steady state error because the system is marginally stable.

\[ \text{\textbf{H.B.}} \]

The steady state errors are calculated for only closed loop stable system.

\[ \text{\textbf{Case (ii): Undamped System (}0 \leq s < 1\text{).}} \]

\[ s_1, s_2 = ? \]

\[ \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \]

\[ s_1, s_2 = -b \pm \frac{\sqrt{b^2 - 4ac}}{2a} \]

\[ = -2\zeta\omega_n \pm \frac{\sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2} \]

\[ s_1, s_2 = -\zeta\omega_n \pm \omega_n \sqrt{s^2 - 1} \]

\[ s_1 : (s + \zeta\omega_n + \omega_n \sqrt{s^2 - 1}) \]

\[ s_2 : (s + \zeta\omega_n - \omega_n \sqrt{s^2 - 1}) \]

\[ \therefore \frac{C(s)}{R(s)} = \frac{\omega_n^2}{(s + \zeta\omega_n + \omega_n \sqrt{s^2 - 1}) (s + \zeta\omega_n - \omega_n \sqrt{s^2 - 1})} \]
Poles lies on left side i.e. \( \mathfrak{S} \).

Real Part:

\[
\gamma = \frac{1}{\omega_n}. 
\]

\( \text{Time constant } \tau = \frac{1}{\omega_n} \).

F.O.O. = \( \omega_a = \omega_n \sqrt{1 - \xi^2} \) and/see.

\[
C(s) = \frac{\omega_n^2}{(s + 3\omega_n)^2 + (\omega_n \sqrt{1 - \xi^2})^2}.
\]

\[
\therefore C(s) = \frac{\omega_n}{\sqrt{1 - \xi^2}} \cdot \frac{\omega_n \sqrt{1 - \xi^2}}{(s + 3\omega_n)^2 + (\omega_n \sqrt{1 - \xi^2})^2}.
\]

\[
C(t) = \frac{\omega_n}{\sqrt{1 - \xi^2}} \cdot e^{-\omega_n t} \sin \omega_n \sqrt{1 - \xi^2} t.
\]

exp. decay & F.O.O.
\[ c(t) \]

\[ t = \frac{1}{\omega n} \]

**Damped oscillation**  
**Underdamped sys.**

\[ \omega_d = \omega_n \sqrt{1 - \xi^2} \text{ rad/sec.} \]

\[ \Rightarrow \text{When } \xi > 0 \Rightarrow 0 < \xi < 1, \text{ the poles lies in the left of s-plane which are complex conjugate. The system stable. The system response is exponential decay per. of oscillations.} \]

\[ \Rightarrow \text{Any system which produced the damped oscillation is called Under-damped system.} \]

\[ \Rightarrow \text{Case (iii): } \xi = 1 \text{ Critical damped:} \]

\[ \frac{c(t)}{\tau_{\text{eff}}} = \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2} \]
\[
\frac{C(s)}{R(s)} = \frac{\omega_n^2}{(s + \omega_n)^2} - \omega_n t.
\]

\[
\therefore C(t) = \omega_n^2 t - e^{-\omega_n t}.
\]

\[
S = -\omega_n - j\omega_n.
\]

\[
T = \frac{1}{\omega_n}.
\]

\[
F(t) = 0.
\]

\[
\text{s-plane}
\]

\[
\text{\textcopyright}
\]

\[
\text{\textcopyright}
\]

\[
C(t)
\]

\[
\text{\textcopyright}
\]

\[
\text{\textcopyright}
\]
when $s = 1$ both the poles are zero.

lies on the negative real axis at the same location the system is stable. The system response is called a critical damped system because it generates critically one damped oscillations.

$\Rightarrow$ The value of the resistance used to get the critical damped nature is called critical resistance.

Case - (iv): $\frac{8}{8} > 1$ : Overdamped System:

$s = s_1, s_2 = -\frac{8}{8} \pm \frac{8}{8} \sqrt{s^2 - 1}$.

$\Rightarrow$

By real part:

$$T = \frac{1}{s an - \frac{8}{8} \sqrt{s^2 - 1}}$$
\[
\Rightarrow \quad (CS) = \frac{\omega n^2}{(s + \varphi_n - \omega n\sqrt{s^2 - 1}) (s + \varphi_n + \omega n\sqrt{s^2 - 1})}
\]

\[
\Rightarrow \quad (CS) = \frac{k_1}{(s + \varphi_n - \omega n\sqrt{s^2 - 1})} - \frac{k_2}{(s + \varphi_n + \omega n\sqrt{s^2 - 1})}
\]

\[
(C(t)) = \frac{- (\varphi_n - \omega n\sqrt{s^2 - 1})t}{k_1 e} - \frac{- (\varphi_n + \omega n\sqrt{s^2 - 1})t}{k_2 e}
\]

D.P. (\(\uparrow\uparrow\))

(\(\uparrow\uparrow\))

\[
\text{Exact Res of D.D system}
\]

\[
\text{Approx Res of D.D system}
\]
When \( |\xi| > 1 \) both the poles lies in the left of S-plane at different location the system is stable. The system response is called over damped system because the system response elements are over comes the damped oscillation.
**Conclusion:**

- When $\xi$ increases from $-1$ to $+1$, the second order system poles path is a circle with a radius of $\omega_n$.

- Radial distance of complex pole is $\xi \omega_n$.

- The value of $\xi$ to the given pole location is:
  - $\xi = 1$
  - $\xi = -1$
  - $|\xi| = 1$
  - None

- When $\xi$ increases from 0 to 1, the poles move towards the left and near to the real-axis. In this case:
  1. Time Constant $\downarrow$
  2. Satellining time $\downarrow$
  3. $\omega_n \downarrow$
  4. As $\omega_n \downarrow$, the time specification $t_d$, $t_r$, $t_p \uparrow$ and the system becomes more relatively stable.
when \( s \) increases from 1 to \( \infty \) then one pole moves towards the origin on the real axis. In this case:

1. \( \tau \uparrow \)
2. \( t_s \uparrow \)
3. damped oscillation becomes 0.
    
    i.e. \( \omega = 0 \).
4. the relative stability of the system decreases.

\[ \Rightarrow \text{Order of the time constant} \]

\[ \Rightarrow \text{Unstable} > \text{Overdamped} > \text{Underdamped} > \text{Critical damped} \]

\[ \frac{1}{\sqrt{\omega_n^2 - \omega_n^2}} \quad \left( \frac{1}{\sqrt{\omega_n^2}} \right) \quad \left( \frac{1}{\omega_n} \right) \]

\( \infty \)

\( \mu \)

\( \mu \)

\( \mu \)

\( \mu \)

(Slow response. & sluggish system)
Unit Step Response:

\[ r(t) = 1 \text{ u}(t) \]

\[ R(s) = \frac{1}{s} \]

\[ R(s) = \frac{s}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)} \]

\[ (c(t)) = \frac{\omega_n^2}{s(s^2 + \omega_n^2)} \]

Case (i): \( \zeta = 0 \): Undamped System.

\[ (c(t)) = \frac{\omega_n^2}{s s^2 + \omega_n^2} \]

\[ (c(t)) = \frac{A}{s} + \frac{B s + C}{s^2 + \omega_n^2} \]

\[ = \frac{1}{s} + \frac{s(-1) + 0}{s^2 + \omega_n^2} \]

\[ (c(t)) = \frac{1}{s} - \frac{s}{s^2 + \omega_n^2} \]

\[ (c(t)) = 1 - \cos(\omega nt) \]

Diagram showing the response of an undamped system with a constant amplitude and frequency of oscillation around the I/P.
Case - (ii) \( \delta > 0, \delta < 1 \) \([0 < \delta < 1]\):

Underdamped system.

\[
C(s) = \frac{\omega_n^2}{s(s + \delta \omega_n - j \omega_n(\sqrt{1 - \delta^2}) (s + \delta \omega_n + j \omega_n \sqrt{1 - \delta^2}))}
\]

\[
\Rightarrow \text{I.L.T to the above } C(s) \text{ is,}
\]

\[
C(t) = 1 - e^{-\delta \omega_n t} \sin \left[ \frac{\omega_n \sqrt{1 - \delta^2} t}{\sqrt{1 - \delta^2}} + \tan^{-1} \left( \frac{\sqrt{1 - \delta^2}}{\delta} \right) \right]
\]

\[
\Rightarrow \tan \theta = \frac{\sqrt{1 - \delta^2}}{\delta}
\]

\[
\Rightarrow \cos \theta = \delta \Rightarrow \theta = \cos^{-1}(\delta)
\]

\[
C(t) = 1 - e^{-\delta \omega_n t} \sin \left( \omega_n t + \tan^{-1} \cos^{-1}(\delta) \right)
\]

\[
\Rightarrow C(t)
\]
Case - III: \( \xi = 1 \) \Rightarrow Critical\ damped.

\[ C(s) = \frac{\omega_n^2}{s(s^2 + 2\omega_n s + \omega_n^2)} = \frac{\omega_n^2}{s(s + \omega_n)^2} \]

\[ = \frac{A}{s} + \frac{B}{s + \omega_n} + \frac{C}{(s + \omega_n)^2} \]

\[ C(\infty) = \frac{1}{s} - \frac{\omega_n}{(s + \omega_n)^2} - \frac{1}{s + \omega_n} \]

\[ \therefore C(t) = \left( 1 - \omega_n (t - e^{-\omega_n t}) - e^{-\omega_n t} \right) \]

![Graph showing critical damped system](image)

Case - IV: \( \xi > 1 \) \Rightarrow Overdamped.

\[ C(s) = \frac{\omega_n^2}{s(s + \omega_n - \omega_n \sqrt{\xi^2 - 1})(s + \omega_n + \omega_n \sqrt{\xi^2 - 1})} \]

\[ = \frac{1}{s} - \frac{k_1}{(s + \omega_n - \omega_n \sqrt{\xi^2 - 1})} - \frac{k_2}{s + \omega_n + (\omega_n \sqrt{\xi^2 - 1})} \]
\[ C(t) = 1 - k_1 e^{-\omega_n \sqrt{s^2 - 1} t} - k_2 e^{-\omega_n \sqrt{s^2 + 1} t} \]

\[ = 1 - \left[ k_1 e^{-\omega_n \sqrt{s^2 - 1} t} + k_2 e^{-\omega_n \sqrt{s^2 + 1} t} \right] \]

\[ c.d. \ (\tau \downarrow) \quad \text{slosh Res.} \quad \text{o.d.} \ (\tau \uparrow) \]

\[ \text{Time Domain Specification:} \]

\[ \Rightarrow \text{For time domain specification select a unit step input and underdamped system.} \]

<table>
<thead>
<tr>
<th></th>
<th>( t_{\tau} )</th>
<th>( ss )</th>
<th>( s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>\textbf{Impulse}</td>
<td>✔️</td>
<td>x</td>
<td>-</td>
</tr>
<tr>
<td>\textbf{Step}</td>
<td>✗</td>
<td>✔️</td>
<td>-</td>
</tr>
<tr>
<td>\textbf{Sump}</td>
<td>✗</td>
<td>✗</td>
<td>x</td>
</tr>
<tr>
<td>\textbf{Parabolic}</td>
<td>✗</td>
<td>✗</td>
<td>x</td>
</tr>
</tbody>
</table>

\[ \Rightarrow \text{For system behaviour Impulse is used.} \]

\[ \Rightarrow \text{For analysis w.r.t. time step is used.} \]
1. $\xi = 0$: Undamped System

2. $\xi = 1$: Critical Damped ($c_{ts} \approx 0.5$ to $35\%$)

3. $\xi > 1$: Overdamped ($c_{ts} \approx 0.1$ to $90\%$)
(4) If $0 < \xi < 1$: Underdamped System.

\[ 0.4 < \xi < 0.7. \]

\[ C(t) \]

\[ t_0 \text{(moderate)} \]

\[ t_5 \text{(moderate)} \]

\[ \star \]

Optimum Value of $\xi$:

\[ t_5 \downarrow t_0 \]

\[ S = 0.1 \]

\[ S = 0.9 \]

\[ S_0, \quad 0.4 < \xi < 0.7 \]
For time domain specification select the underdamped system because.

1. If select the undamped system the rise time is very small. The settling time is infinity whereas if select the critical damping system rise time is large but settling time is very small.

2. If select the overdamped system the rise time is large and setting time also large.

Practically for any system, required smallest rise time and smallest settling time.

In underdamped system we can get the moderate values of rise time and settling time.

3. In underdamped system the best range of $\xi$ is $0.4 < \xi < 0.7$.

4. When $0 < \xi < 1$, the unit step response of the system is:
\[ C(t) = \left[ 1 - \frac{-\frac{\varepsilon \omega nt}{N^{1-\varepsilon^2}}}{\sin \left( \omega_n \sqrt{1-\varepsilon^2} + \tan^{-1} \left( \frac{\sqrt{1-\varepsilon^2}}{\varepsilon} \right) \right)} \right] \]

\[ \omega_d = \omega_n \sqrt{1-\varepsilon^2} \]

---

**Delay time**: \( t_d \):

- It is the time required for the response to rise from 0 to 0.5 \( \mu \) of the final value.
\[ t_d = \frac{1 + 0.75}{\omega_n} \text{ sec.} \]

(2) **Rise time (\( t_r \))**: It is the time required for the response to rise from 0 to 100\% of its final value for underdamped system, 5\% to 95\%; for critically damped system and 10\% to 90\% for overdamped system.

\[ t_r = \pi - \tan^{-1} \frac{\sqrt{1 - \xi^2}}{\xi} / \omega_d \]

\[ t_r = \pi - \cos^{-1} (\xi) / \omega_d \]

(3) **Peak time (\( t_p \))**: It is the time required for the system to rise from 0 to peaks of the time response.
\[ t_p = \frac{n\pi}{\omega_d} \]

\( n = 1 \rightarrow \text{by default} \rightarrow 1^{st} \text{ peak.} \)

\[ t_p = \frac{\pi}{\omega_d} \]

\( \rightarrow \text{For 2nd peak,} \)

\[ t_p = \frac{2\pi}{\omega_d} \]

\( \rightarrow \text{For 2nd valley,} \quad t_p = \frac{2\pi}{\omega_d} \)

4. **Peak-overshoot (MP):**

\[ M_p = C(t_p) - C(\infty) \]

5. \( \% \) of **Peak-overshoot (\%MP):**

\( \Rightarrow \text{It is the normalized difference between time response peak to steady state.} \)

\[ \%M_p = \frac{C(t_p) - C(\infty)}{(C(\infty))} \times 100\% \]

\( \Rightarrow \%M_p = [(C(t_p) - 1)] \times 100\%. \)
\[ M_p = \frac{-\pi s}{\sqrt{1-s^2}} \times 100 \% \]

\[ M_p = \frac{-\pi s}{\sqrt{1-s^2}} \times 100 \% \]

\[ \Rightarrow \text{The } n \text{-value is similar to peak time.} \]

\[ \Rightarrow \text{The underdamped to the first valley point is.} \]

\[ M_V = \frac{-2\pi s}{\sqrt{1-s^2}} \times 100 \% \]

\[ \Rightarrow \text{Settling Time } (T_s): \]

\[ \Rightarrow \text{It is the time required for the response to rise from 0 to specified tolerance band usually } \pm 2\%. \]

\[ T_s = \frac{2}{\delta \omega n} \text{ sec.} \]

\[ T_s = \frac{4}{\delta \omega n} \text{ sec.} \rightarrow \text{default.} \]

\[ T_s = \frac{5}{\delta \omega n} \text{ sec. (SS value).} \]

\[ \Rightarrow \text{Time Period of Oscillation:} \]

\[ \Rightarrow \text{It is the time required to complete one cycle.} \]
\[ T_{osc} = \frac{2\tau}{\omega_d} = 2\tau_p. \]

\[ N = \frac{t_s}{2\pi/\omega_d} = \frac{t_s \cdot \omega_d}{2\pi}. \]

\[ N = \frac{t_s}{2\tau_p}. \]

**Q:** The unit step response of the system is shown in Fig. and Find the following factors.  
1. \( \gamma \)  
2. \( t_a \)  
3. \( t_s \)  
4. \( \tau_p \) and \( \tau \).

**Diagram:**

\[ C(t) \]

\[ 1 \]

\[ t = 10 \text{ sec} \]

\[ t_s = 0.02 \text{ sec} \]

\[ t_s = \frac{4\gamma}{1} \quad (\because \pm 2\mp). \]

\[ t_s = 4\gamma \Rightarrow \gamma = \frac{t_s}{4} \]

\[ \tau = t_{s/4} \approx 2.5 \text{ sec}. \]
The standard form of the unit step response is:

\[ c(t) = k(1 - e^{-t/\tau}) \]

S.S. Value

\[ \therefore c(t) = (1 - e^{-t/2.5}) \]

1. \( \tau = 2.5 \text{ sec.} \)
2. \( td \)
   - \( \Rightarrow \) at \( t = td \) \( \Rightarrow c(t) = 0.5 \)
   - \( -td/2.5 \)
   - \( 0.5 = 1 - e^{-td/2.5} \)
   - \( e^{-td/2.5} = 0.5 \)
   - \( td = 1.733 \text{ s} \)
3. \( tf \)
   - \( \Rightarrow \) at \( kx/100 \) \( \Rightarrow c(t) = 0.1 \)
   - \( -tf/2.5 \)
   - \( 0.1 = 1 - e^{-tf/2.5} \)
   - \( tf = 0.263 \text{ s} \)
\[ t = t_{x2} \Rightarrow c(t) = 0.9, \quad -t_{x2}/2.5 \]
\[ 0.9 = 1 - e^{-t_{x2}/2.5} \quad \Rightarrow \quad t_{x2} = 5.365 \]

\[ t_x = t_{x2} - t_{x1} = 2.2\gamma \]

\[ t_x = 2.2\gamma \]

\[ t_x = 2.2 \times 2.5 \]

\[ t_x = 5.5 \text{ sec} \]

\[ \Rightarrow 4) \quad t_p \& M_p. \]

There is no peaks are exist hence no peak time and no peak overshoot.

The impulse response of a system is \( c(t) = k \cdot e^{-3t} \cdot \sin(\omega t) \). Find the following factors:

1. \( \omega \)
2. \( \omega_n \)
3. \( s \)
4. \( M_p \)
5. \( t_a \)
6. \( t_o \) and \( \Theta_t \).

\[ c(t) = k \cdot e^{-3t} \cdot \sin(\omega t) \]
1. \( \gamma = \frac{1}{3} \text{ sec.} \)

\[ \omega_0 = 4 \quad \text{and} \quad \frac{1}{\sec} \]

\[ \omega_n = \sqrt{\frac{3^2 + 4^2}{3}} \]

\[ \Rightarrow \quad \omega_n = 5 \quad \text{and} \quad \frac{1}{\sec} \]

2. \( \omega_n = 5 \quad \text{and} \quad \frac{1}{\sec} \)

3. \( \xi = \frac{4}{5} \)

\[ (\frac{4}{5})^2 = 1 - \xi^2 \]

\[ 1 - \frac{16}{25} = \xi^2 \]

\[ \xi = \frac{3}{5} \]

\[ \xi = 0.6 \]

4. \( M_p := -\frac{11.5}{\sqrt{1 - \xi^2}} \)

\[ \therefore M_p = e \times 100 \quad \therefore \]

\[ -\frac{3.14 \times 0.6}{\sqrt{1 - 0.36}} \]

\[ = e \times 100 \quad \therefore \]

\[ \Rightarrow \quad \therefore M_p = 9.5 \quad \therefore \]

5. \( t_d := \quad t_d = \frac{1 + 0.3\xi}{\omega_n} \)
\[ t_d = 0.284 \text{ sec} \]

6. \( t_8 : \)
\[
\begin{align*}
  t_r &= \frac{\pi - \cos^{-1}(0.6)}{\omega_d} \\
  &= \frac{3.14 - \cos^{-1}(0.6)}{\omega_d} \\
  &= \frac{3.14 - 53.13^\circ}{4} \\
  &= \frac{3.14 - \left(\frac{53.13^\circ \times \pi}{180^\circ}\right)}{4} \\
  \therefore \quad t_8 &= 0.553 \text{ sec}
\end{align*}
\]

7. \( t_p : \)
\[
\begin{align*}
  t_p &= \frac{\pi}{\omega_d} = \frac{3.14}{4} \\
  \therefore \quad t_p &= 0.785 \text{ sec}
\end{align*}
\]

8. The unit step response of the system is shown in figure. Find the following factors:
   ① \( M_p \)  ④ \( \omega_n \)
   ② \( \tau \)  ⑤ delay time.
   ③ \( s \)  ⑥ \( t_e \)
   ⑧ \( \text{OLTF} \)
   ⑨ \( \text{CLTF} \), assume UFB system.
1. \( M_p: \)

\[ C(t_p) = 1.25 \quad , \quad C(\infty) = 1 \]

\[ M_p = C(t_p) - C(\infty) = 1.25 - 1 = 0.25 \]

\[ M_p = 0.25 \]

2. \( \therefore M_p: \)

\[ \therefore M_p = \frac{C(t_p) - C(\infty)}{C(\infty)} \times 100 \]

\[ \therefore M_p = 25 \%

3. \( \lim: \)

\[ -\frac{\pi \xi}{\sqrt{1-\xi^2}} \]

\[ \therefore M_p = e \times 100 \]

\[ \therefore \frac{25}{100} = e \]

\[ \therefore -\frac{\pi \xi}{\sqrt{1-\xi^2}} = -1.386 \]
\[
\frac{\xi}{\sqrt{1-\xi^2}} = 0.441
\]

\[
\therefore \quad \xi^2 = (0.195)(1-\xi^2).
\]

\[
\therefore 1.195 \xi^2 = 0.195
\]

\[
\Rightarrow \quad \xi = 0.404.
\]

4 \quad w_n:

\[k = \text{V sec}.
\]

\[k \omega = \text{max}.
\]

\[\alpha = \frac{\alpha}{\omega n}\]

\[\omega n \approx \frac{\omega n}{8} \Rightarrow \omega n = \frac{\omega n}{8}.
\]

\[
\therefore \quad t_p = \frac{\pi}{\omega a}.
\]

\[
\therefore \quad \omega a = 3.14.
\]

\[
\Rightarrow \quad \omega a = \omega n \sqrt{1-\xi^2}
\]

\[
\therefore \quad \omega n = \frac{3.14}{\sqrt{1-0.16}}
\]

\[
\Rightarrow \quad \omega n = 3.43 \text{ rad/sec}
\]

5 \quad t_d:

\[
\therefore \quad t_d = \frac{1 + 0.35}{\omega n}
\]

\[
= \frac{1 + (0.7)(0.4)}{3.43}
\]

\[
\therefore \quad t_d = 0.373 \text{ sec}
\]
6. \[ t_r = \frac{\pi - \cos^{-1}(0.4)}{\omega_d} \]

\[ = \frac{3.14 - \cos^{-1}(0.4)}{3.14} \]

\[ \therefore t_r = 0.63 \text{ sec} \]

7. \[ t_s = 4\gamma = \frac{4}{\frac{1}{2} \omega_n} \]

\[ \therefore t_s = \frac{4}{0.4 \times 3.43} \]

\[ \therefore t_s = 2.915 \text{ sec} \]

8. \[ G(s) = \frac{\omega_n^2}{s(s + 2.5\omega_n)} \]

\[ = \frac{(3.43)^2}{s(s + 2(0.4)(3.43))} \]

\[ G_r(s) = \frac{11.765}{s(s + 2.344)} \]

9. \[ \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2.5\omega_n s + \omega_n^2} \]

\[ \therefore \frac{C(s)}{R(s)} = \frac{11.765}{s^2 + 2 \times 2.344 s + 11.765} \]
@ Find the I.M.P to the following system to the unit step input.

\[ \frac{C(s)}{R(s)} = \frac{25}{s^2 + 25} \]

\[ \text{So., } \therefore M_p = \frac{100}{1} \]

\[ \therefore \boxed{M_p = 100 \text{ ft.}} \]

@ Find \( M_p \) of \( \frac{C(s)}{R(s)} = \frac{100}{s^2 + 20s + 100} \)

\[ \text{Hence, } \omega_n^2 = 100 \]

\[ \Rightarrow \boxed{\omega_n = 10 \text{ rad/sec}} \]

\[ 25\omega_n = 25 \times 10 \]

\[ \boxed{\zeta = \frac{1}{5}} \rightarrow \text{CD} \]

\[ \frac{-\pi \zeta}{\sqrt{1-\zeta^2}} \]

\[ \therefore M_p = \frac{100}{\pi} \times 100 \text{ ft.} \]

\[ \therefore M_p = 100 \text{ ft.} \]

\[ \boxed{M_p = 0 \text{ ft.}} \]
Note: When $s$ increases from 0 to 1,

- $M_p$ decreases from 100% to 0%

When $s \geq 1$, $M_p = 0\%$ because no oscillation exists in the system.

Find the variation in time domain specification to the given poles path in the $s$-plane.

As real part is constant, time constant is constant hence settling time is constant.

As imaginary part increases, the damped oscillation $\omega_d$ increases. As $\omega_d$
increases the time specification to, to a \( \frac{1}{\omega} \) decreases.

\[ c(t) \]

\[ \text{Optimum value of } \frac{1}{\omega} = 5 \% \text{ to } 25 \%. \]

\[ \Rightarrow \] As the inclination of the pole \( \theta \) increases, the damping ratio \( \xi \) decreases. Hence, the \% of \( mp \) increases.

\[ \Rightarrow \] The larger \( mp \) make the system less RS & more oscillatory.

\[ \Rightarrow \] The optimum range of the \% \( mp \) is \( 5 \% \text{ to } 25 \% \).

\[ \Rightarrow \] If the peak overshoot is more than \( 25 \% \), the system is less relative stable.

\[ \Rightarrow \] If the peak overshoot is \( mp < 5 \% \), the system is slow response.
[Diagram showing time-domain analysis]

**Problem:** Find the variation in time domain specification to the given pole path in S-plane.

**Solution:**
- \( \omega_n \) increases.
- \( \zeta \) decreases slightly.
- More RL (5)
⇒ Pole moving towards the left side
and $\xi_2 \approx \gamma$ both decreases.

⇒ Imaginary part is constant.

So, $\omega_d = \text{constant}.$

$$t_p = \frac{\pi}{\omega_d} = \text{Constant}.$$  

⇒ $t_p = \text{constant}.$

⇒ As imaginary part is constant
the damped oscillations $\omega_d$ constant
but there exist a slight variation in
$\xi_2$ and $\gamma.$

⇒ As the inclination at the pole $\theta$
decreases the damping action increases,
hence the $/\lim_{t \to \infty}$ decreases.
become more stable.
Find the variation in the time specification when location of poles moves or change as shown in fig.

\[ s \text{-plane} \]

\[ \theta = \text{constant} \]
\[ \xi = \text{constant} \]
\[ \Rightarrow M_p = \text{constant} \]
\[ \omega_d \uparrow \]
\[ \omega_n \uparrow \]

\[ \tau_d \downarrow \]
\[ \tau_r \downarrow \]
\[ \tau_p \downarrow \]
\[ \tau_s \uparrow \]

\[ \Rightarrow \] As the inclination of the pole \( \theta = \text{constant} \)
the damping ratio \( \xi \) is constant and hence \( \Rightarrow M_p \) is also constant.

\[ \Rightarrow \] As the poles moves towards the left, time constant \( \tau \) decreases hence settling time decreases.

\[ \Rightarrow \] As imaginary part is increases damped oscillation \( \omega_d \) increases hence \((\tau_d, \tau_r, \tau_p) \downarrow\) hence \( \omega_n \downarrow \).
Find the time domain specification of

\[ G(s) = \frac{25}{s(s+4)} \]

So:

\[ \frac{dG}{ds} = \frac{25}{s(s+4)} \]

\[ \Rightarrow \omega_n^2 = 25 \]

\[ \omega_n = 5 \text{ rad/sec} \]

\[ \Rightarrow 2\zeta \omega_n = 4.2 \]

\[ \zeta = \frac{2}{5} = 0.4 \]

\[ \omega_d = \omega_n \sqrt{1 - \zeta^2} \]

\[ = \frac{5 \sqrt{1 - 0.16}}{4.58 \text{ rad/sec}} \]

\[ t_d = \frac{1 + 0.3\zeta}{\omega_n} \]

\[ t_d = 1 + \frac{0.3 \times 0.4}{5} = 0.25s \]

\[ t_\theta = \frac{\pi - \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}}{\omega_d} \]

\[ = \frac{\pi - \cos^{-1} \zeta}{\omega_d} \]

\[ t_\theta = \frac{3.14 - \cos^{-1} 0.4}{4.58} \]

\[ t_\theta = 0.4326 \text{ sec} \]
\[ \Rightarrow T_0 = \frac{\pi}{\omega_d} = \frac{3.14}{4.58} \]

\[ \therefore T_0 = 0.686 \text{ sec} \]

\[ \Rightarrow M_p = \frac{\pi^2 / (1 - \frac{c^2}{2})}{3.14^2 / 10.84} \times 100 \% \]

\[ = C \times 100 \% \]

\[ M_p = 25.4 \% \]

\[ \Rightarrow t_s = 4\gamma = \frac{4}{5\omega_n} \]

\[ t_s = \frac{4}{0.4 \times 5} \]

\[ \therefore t_s = 2 \text{ sec} \]

---

**CA: Repeat the above problem,**

\[ R(s) \]

\[ \times \]

\[ \frac{20}{s^2 + 8s + 5} \]

\[ \rightarrow C(s) \]

\[ \text{Sum:} \quad G_c(s) = \frac{20}{s^2 + 8s + 5} \]

\[ = \quad (s^2 + 8s + 5) \]

\[ \therefore G(s) = \frac{20}{s^2 + 8s + 5} \]
\[ \frac{C(s)}{R(s)} = \frac{20}{s^2 + 5s + 25} \]

\[ \frac{C(s)}{R(s)} = \frac{20}{25} \left[ \frac{25}{s^2 + 5s + 25} \right] \]

→ This change effect the ss value but do not effect the T.P. specification.

\[ \omega_n^2 = 25 \]
\[ \omega_n = 5 \text{ rad/sec} \]

\[ \tau = \frac{1}{5 \omega_n} = \frac{1}{5 \times 0.5} \]
\[ \tau = 0.4 \text{ sec} \]

\[ t_s = 4\tau \]
\[ t_s = 1.6 \text{ sec} \]

\[ t_d = \frac{1 + 0.75}{\omega_n} = \frac{1 + (0.7 \times 0.5)}{5} \]
\[ t_d = 0.27 \text{ sec} \]

\[ t_\theta = \frac{\pi - \cos^{-1}(0.5)}{\omega_d} \]
\[ t_\theta = \frac{\pi - \cos^{-1}(0.5)}{4.33} \]

2.5 \omega_n = 5
2 \times 3 \times \frac{1}{2} = 5
\[ \frac{5}{5} = 0.5 \]

\[ \omega_d = \omega_n \sqrt{1 - \delta^2} = 5 \sqrt{1 - 0.25} = 4.33 \text{ rad/sec} \]

\[ t_d = 0.483 \text{ sec} \]
\[ t_p = \frac{\pi}{\omega_d} = \frac{3.14}{4.33} = 0.725 \text{ sec} \]

\[ M_p = e^{-\frac{\pi \xi}{\sqrt{1-\xi^2}}} \times 100 \% \]

\[ = e^{-3.14 \times 0.5 / \sqrt{0.75}} \times 100 \% \]

\[ M_p = 16.31 \% \]

\[ \Rightarrow \text{the unit step response to the above system is,} \]

\[ c(t) = 1 - \frac{e^{-5\sqrt{t}}}{{\sqrt{1-\xi^2}}} \sin \left( \omega_d t + \cos^{-1} \xi \right) \]

\[ c(t) = 1 - \frac{-2.5t}{\sqrt{0.75}} \sin \left( 4.33t + 60^\circ \right) \]

\[ \Rightarrow \text{Variation in time domain specification w.r.t. } \xi \text{ (} \omega_n = \text{ constant).} \]

\[ \Rightarrow \text{As } \xi \text{ increases from 0 to 1, the poles move towards the left and near to the real axis.} \]

\[ \Rightarrow \text{In this case} \]

1. \( T \downarrow \]
2. \( L \uparrow \]
3. \( \omega_d \downarrow \]
As \( \omega_n \) decreases the time domain specification (\( t_d, t_{r}, t_{s} \)) increases.

As \( \xi \) increases from 0 to 1, \( \omega_n \) decreases and the system become more relatively stable.

Find the T.D. specification to the following system

\[
\frac{d^2y}{dx^2} + 4 \frac{dy}{dt} + 8y = 8x.
\]

Result:

\[
\frac{C(s)}{R(s)} = \frac{Y(s)}{X(s)} = \frac{8}{s^2 + 4s + 8}
\]

\( \Rightarrow \) \( \omega_n^2 = 8 \)

\( \omega_n = 2\sqrt{2} \) rad/sec.

\( \omega_n = 2.83 \) rad/sec

\( \xi = 0.707 \)

\( \Rightarrow \) \( \gamma = \frac{1}{\xi \omega_n} = \frac{1}{\sqrt{2} \times 2.83} = \frac{1}{2.83} = 0.4 \)

\( \gamma = 0.5 \) sec

\( \Rightarrow \) \( t_s = 4\gamma = 4 \times 0.5 \)

\( t_s = 2 \) sec
\[ t_d = \frac{1 + 0.75}{\cos \theta} \]
\[ = 1 + \frac{(0.7 \times 0.703)}{2.83} \]
\[ t_d = 0.53 \text{ sec} \]

\[ t_8 = \frac{\pi - \cos^{-1} 0.703}{\omega_d} \]
\[ t_8 = \frac{\pi - \cos^{-1} 0.703}{2} \]
\[ t_8 = 1.177 \text{ sec} \]

\[ \omega_d = \omega_n \sqrt{1 - s^2} \]
\[ \omega_d = 2 \text{ rad/sec} \]

\[ t_p = \frac{\pi}{\omega_d} \]
\[ t_p = 3.14/2 \]
\[ t_p = 1.57 \text{ sec} \]

\[ M_p = \frac{-11.5}{\sqrt{1 - s^2}} \times 100 \% \]
\[ M_p = \frac{-3.14 \times 0.703}{\sqrt{1 - 0.703^2}} \]
\[ M_p = 4.33 \% \]
Steady State error:

\[ \lim_{t \to \infty} e(t) = \text{Steady State error} \]

\[ \Rightarrow \text{The error is nothing but deviation of the output from the input.} \]

\[ \Rightarrow \text{Steady State error is the error at } t \to \infty. \]

\[ E_{ss} = \lim_{t \to \infty} e(t) \]

\[ C : \text{Final value theorem} \]

\[ E_{ss} = \lim_{s \to 0} sE(s) \]

\[ \therefore E_{ss} = \lim_{s \to 0} sE(s) \]

\[ \Rightarrow \text{Consider the UFB system as shown in figure:} \]

\[ R(s) \quad + \quad E(s) \]

\[ \times \quad G(s) \quad \rightarrow \quad C(s) \]

\[ \Rightarrow E(s) = R(s) - C(s) \]

\[ C(s) = G(s) \cdot E(s) \]

\[ \therefore E(s) = R(s) - G(s) \cdot E(s) \]

\[ \Rightarrow \frac{E(s)}{R(s)} = \frac{1}{1 + G(s)} \]
\[ e_{ss} = \lim_{s \to 0} \frac{SR(s)}{1 + CR(s)} \]

The steady state errors depend on two factors:
1. Type of Input.
2. Type of System.

The steady state errors are calculated to only CL stable system.

The steady state errors are valid only for UFB system.

If non-unit FB system is given it should be converted into UFB.

* Type of Input:

<table>
<thead>
<tr>
<th>I/p ( \delta(t) )</th>
<th>Step ( A u(t) )</th>
<th>Ramp ( A t u(t) )</th>
<th>Parabolic ( A t^{2/2} u(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e_{ss} )</td>
<td>( \frac{A}{1 + K_p} )</td>
<td>( \frac{A}{K_v} )</td>
<td>( \frac{A}{K_a} )</td>
</tr>
</tbody>
</table>

Error Constant:
- \( K_p \): Position err\( \lim_{s \to 0} c(r(s)) \)
- \( K_v \): Velocity (ms) \( \lim_{s \to 0} s c(r(s)) \)
- \( K_a \): Acceleration (ms²) \( \lim_{s \to 0} s^2 c(r(s)) \)
Type of System:

The standard form of the system is:

$$G_c(s) = \frac{k (1 + s\tau_1) (1 + s\tau_2) \ldots}{s^n (1 + s\tau_a) (1 + s\tau_b) \ldots}$$

Type = n system.

Consider the step input and the different type of the system.

0. Step ⇒ \( \left( t_0, u(t) \right) \).

$$e_{ss} = \frac{A}{1+K_p}$$

$$K_p = \lim_{s \to 0} G_c(s).$$

1. Type = 0:

$$K_p = \lim_{s \to 0} \frac{k (1 + s\tau_1) (1 + s\tau_2) \ldots}{s^n (1 + s\tau_a) (1 + s\tau_b) \ldots} = K$$

$$e_{ss} = K \frac{A}{1+K_p} = \frac{A}{1+K}$$

$$e_{ss} = \frac{A}{1+K} = \text{constant}$$
(2) Type - 1:

\[ k_p = \lim_{s \to 0} \frac{K}{s} \frac{(1 + SY_1)(1 + SY_2) \ldots}{s^1 (1 + SY_a)(1 + SY_b) \ldots} \]

\[ k_p = \infty \quad \Rightarrow \quad e_s = \frac{A}{1 + \infty} = 0 \]
\[ \Rightarrow e_{ss} = 0 \]

(3) Type - 2:

\[ k_p = \lim_{s \to 0} \frac{K}{s^2} \frac{(1 + SY_1)(1 + SY_2) \ldots}{s^2 (1 + SY_a)(1 + SY_b) \ldots} \]

\[ k_p = \infty \quad \Rightarrow \quad e_{ss} = 0 \]

⇒ The s.s. errors are required to calculate only in 3 cases:

i.e ① Type - 0 & step input (t°).
② Type - 1 & ramp input (t').
③ Type - 2 & parabolic input (t²).

⇒ Remain at all the cases the steady state error either become zero or infinity.
**Find \( e_{ss} \) to the given unity FB system**

\[ u(s) = \frac{10(s+1)}{s^2(s+2)(s+10)}, \quad h(s)=1. \]

**to the following input**

\[ v(t) = (10 + 2t + \frac{1}{2}t^2) \cdot u(t). \]

**So:**

\[ R(s) = \frac{10}{s} + \frac{2}{s^2} + \frac{1}{s^3}. \]

\[ e_{ss} = \lim_{s \to 0} \frac{S \cdot R(s)}{1 + u(r(s))}. \]

\[ e_{ss} = \lim_{s \to 0} \frac{S \left( \frac{10}{s} + \frac{2}{s^2} + \frac{1}{s^3} \right)}{1 + \frac{10(s+1)}{s^2(s+2)(s+10)}}. \]

\[ e_{ss} = \lim_{s \to 0} \frac{(s^2 + 2s + 1)(s+2)(s+10)}{s^2(s+2)(s+10) + 10(s+1)} \]

\[ = \frac{(0+2)(0+10)}{0+10(0+1)} = \frac{10}{40} = 0.2. \]
Method-2: Compare type and input.

\[ \text{Type } \ddot{e} \dot{1} p \quad \text{ess} \]
\[ 2 > 0 \quad \Rightarrow \quad 0 + \]
\[ 2 > 1 \quad \Rightarrow \quad 0 + \frac{A}{k} \]
\[ 2 = 2 \quad \Rightarrow \quad \frac{1}{10x1} + \frac{1}{10x2} = 2 \]

\[ \text{ess} = 2 \]

Find the ess to the following inputs to the given UFB system.

\[ G(s) = \frac{10}{s(s+5)} \]
\[ K(s) = 1 \]

1. \(10u(t)\)
2. \(10t^2u(t)\)
3. \(10t^2u(t)\)
4. \((1+t+2t^2)u(t)\)
5. \((1+t+2t^2)u(t)\)

So: \[ G(s) = \frac{10}{s(s+5)} \]

Type-1:

1. Type > (\(\ddot{e} \dot{1} p = 0\))

So, \[ \text{ess} = 0 \]
2. Type = (\(\lambda_1 p = 1\))

\[
\text{ess} = \frac{A}{K} = \frac{1.55}{2.10/3}
\]

\[\text{ess} = 5\]

3. Type < (\(\lambda_1 p = 2\))

\[\text{ess} = \infty\]

4. \((1+t)u(t) = u(t) + tu(t)\)

\[
\Downarrow \quad \text{ess} = 0 \quad \Downarrow \quad \text{ess} = \frac{A}{K} = \frac{1}{1.015}
\]

\[\text{ess} = 0.5\]

5. \((1+t+t^2)u(t)\)

\[
\Downarrow \quad \text{ess} = 0 \quad \Downarrow \quad \text{ess} = 0.5 \quad \Downarrow \quad \text{ess} = \infty
\]

\[\text{ess} = \infty\]

[Q] Repeat the above problem 200

\[C_{CS(1)} = \frac{(S+1)}{s^2(s+5)(s+10)}\]

\[
\text{Solv.} \quad \text{Type} = 2:
\]

1. Type > (\(\lambda_1 p = 0\))

\[\text{ess} = 0\]
2. \( \text{Type} > (\dot{\lambda}_{10} = 1) \)
   \[ \Rightarrow \quad e_{ss} = 0 \]

3. \( \text{Type} = (\dot{\lambda}_{10} = 2) \)
   \[ e_{ss} = A/k = \frac{2 \times 10}{1 \times 10} = 2 \quad (\text{given}) \]
   \[ (1^2 + 2^2) u(t) \]
   \[ = u(t) + t \cdot u(t) + t^2 u(t) \]
   \[ \downarrow \quad \downarrow \quad \downarrow \]
   \[ 0 \quad 0 \quad e_{ss} = 100 \]
   \[ \therefore \quad e_{ss} = 100 \]

4. \( \text{Type} > \dot{\lambda}_{10} \)
   \[ \Rightarrow \quad e_{ss} = 0 \]

5. \( \frac{1}{s^2(s+5)(s+10)} \quad H(s) = 1 \)

Sum:
- The above system is unstable,
- \( CLTF = \frac{C_r}{1 + \alpha} = \frac{1}{s^4 + 1s^3 + 5s^2 + 1} \)
  \[ \downarrow \quad s^1 \text{ missing} \rightarrow \text{US} \]

\[ \Rightarrow \quad \text{The } e_{ss} \text{ are calculated to only CLTF} \]
- Stable System.
Note: Before calculating $\zeta$ and $\omega_n$, observe the options. If any one of the options is 'none' then verify the CLTF system stability by using the RH criteria.

(a) Calculate the $\zeta$ to the given UFB system to the unit step input.

\[
\frac{15}{s+1} \times \frac{3}{s+15} = \frac{45}{(s+1)(s+15)}
\]

\[
\Rightarrow \text{(Type = 0) = (Type = 0)}
\]

\[
SSE \quad \zeta_s = \frac{A}{1+k} = \frac{1}{1 + \frac{45}{15}}
\]

\[
\Rightarrow \zeta_s = 0.25
\]

Note: $\zeta_s$ are calculated to only UFB system by using CLTF i.e. $C(s)$, $\zeta_{s1} = 1$. 
Method 2:

$\Rightarrow$ If any Bo (or) Separation is given (or) Non-UPB is given, then

$$e_{ss} = \lim_{t \to \infty} [\theta(t) - C(t)]$$

$\Rightarrow e_{ss} = \lim_{s \to 0} s \left[ R(s) - C(s) \right].$

$$= \lim_{s \to 0} s \cdot R(s) \left[ 1 - \frac{C(s)}{R(s)} \right].$$

$$e_{ss} = \lim_{s \to 0} s \cdot R(s) \left[ 1 - \text{CLTF} \right].$$

So, $\text{CLTF} = \frac{45}{s^2 + 16s + 60}$

$\therefore e_{ss} = \lim_{s \to 0} s \cdot \left( \frac{1}{8} \right) \left( 1 - \frac{45}{s^2 + 16s + 60} \right).$

$$= 1 - \frac{45}{60}$$

$$= 1 - \frac{3}{4}$$

$$= \frac{1}{4}$$

$\Rightarrow e_{ss} = 0.25$
The CLTF of UFBs

\[ CR(s) = \frac{K}{s(s+1)(s+2)} \]

The value of \( K \) to get the s.s. error 0.1 to the unit damp 11\( \% \) is \(-8\). 

**Soln:**

Type = 11%

\[ \Rightarrow ES = \frac{A}{K} \]

\[ :: 0.1 = \frac{A}{K/(s+2)} \]

\[ :: K = \frac{0.1}{0.1} = 20 \]

For the system shown in Fig. the s.s. olp is 2. for unit step input and system time constant is 0.4 sec. The values of \( k_1 \) & \( k_2 \) are ?

\[ R(s) \]

\[ \times \]

\[ \frac{k_1}{s} \]

\[ \frac{k_2}{s} \]

\[ C(s) \]

\[ \text{Soln: } \]

\[ \text{CLTF} = \frac{C(s)}{R(s)} = \frac{k_1/s}{1 + \frac{k_1k_2}{s}} = \frac{k_1}{s + k_1k_2} \]
\[ \tau = \frac{1}{k_1k_2} = 0.4 \]

Now, \( C_{ss} = C(t) = C(\infty) = 2 \).

\[ C(\infty) = \lim_{t \to \infty} C(t) = \lim_{s \to 0} s \cdot R(s) \cdot \frac{k_1}{s + k_1k_2} \]

\[ 2 = \lim_{s \to 0} s \cdot R(s) \cdot \frac{k_1}{s + k_1k_2} \]

\[ 2 = \frac{k_1}{k_1k_2} \]

\[ k_2 = 0.5 \]

Now, \( k_1 \cdot k_2 = \frac{1}{0.4} \)

\[ k_1 = \frac{1}{0.4 \times 0.5} \]

\[ k_1 = 5 \]

**Q1** For the system shown in figure

the natural freq. of osc. is 4 rad/s
and damping ratio is 0.7. The values of \( k \) & \( A \) are?

**Soln:** \( \omega_n = 4 \text{ rad/s, } \zeta = 0.7 \)
\[ C_{LTF} = \frac{C(s)}{R(s)} = \frac{K}{s(s+2)} + \frac{KC(s+\alpha)}{s(s+2)} \]

\[ = \frac{K}{s^2 + 2s + K + Kas} \]

\[ \therefore \frac{C(s)}{R(s)} = \frac{K}{s^2 + (2+Ka)s + K} \]

\[ \Rightarrow \omega_n^2 = K \]

\[ \boxed{K = 16} \]

\[ 2\zeta \omega_n = 2 + Ka \]

\[ \therefore 2\times 0.7 \times 4 = 2 + Ka \]

\[ 3.6 = Ka \]

\[ \Rightarrow \alpha = \frac{3.6}{16} = \frac{9}{40} \approx 0.225 \]

A Control System describe by following differential eqn.

\[ \frac{d^2y}{dt^2} + 5 \frac{dy}{dt} + 5y = 10(1 - e^{-2t}) \]

The response at \( t \to \infty \).
\[ (s^2 + 5s + 5) Y(s) = 10 \left( \frac{1}{s} - \frac{1}{s+2} \right) X(s). \]

\[
\begin{align*}
\text{CLTF} \quad \frac{Y(s)}{X(s)} &= \frac{20}{s(s+2)} \\
&\quad \left( s^2 + 5s + 5 \right) \\
\frac{Y(s)}{X(s)} &= \frac{20}{s(s+2)(s^2 + 5s + 5)}.
\end{align*}
\]

\[ Y(\infty) = \lim_{t \to \infty} y(t) \quad X(s) = 1 \]

\[ Y(s) = \lim_{s \to 0} s \cdot Y(s). \]

\[ = \lim_{s \to 0} s \cdot \frac{20}{s(s+2)(s^2 + 5s + 5)} \]

\[ = \frac{20}{2 \times 5} \]

\[ Y(\infty) = 2 \]

\[ \text{Steady State error} \quad \text{to the} \]

\[ \text{Disturbance} \quad \text{lip:} \quad D(s) \]

\[ R(s) \]

\[ X \]

\[ C_{r_1} \]

\[ + \]

\[ X \]

\[ C_{r_2} \]

\[ C(s) \]

\[ \]
1) $e_{ss}$ due to $R(s)$:

$$\frac{E(s)}{R(s)} = \frac{1}{1 + cr_1 \cdot cr_2}.$$ 

$\therefore e_{ss_1} = \lim_{s \to 0} s \cdot \frac{R(s)}{1 + cr_1 \cdot cr_2}.$

2) $e_{ss}$ due to $D(s)$:

$\therefore \Phi \frac{E(s)}{D(s)} = \frac{-cr_2}{1 + cr_1 \cdot cr_2}.$

$\therefore e_{ss_2} = \lim_{s \to 0} s \cdot \frac{-cr_2 \cdot D(s)}{1 + cr_1 \cdot cr_2}.$

$\therefore e_{ss} = \lim_{s \to 0} s \left[ \frac{R(s) - cr_2 \cdot D(s)}{1 + cr_1 \cdot cr_2} \right].$
Find the ess due to the step input and step disturbance to the following system:

\[
\begin{align*}
\text{So}13: & \\
e_{ss} &= \lim_{s \to 0} s \left[ \frac{R(s) - a_2(s) \cdot B(s)}{1 + a_1(s) \cdot a_2(s)} \right] \\
&= \lim_{s \to 0} s \left[ \frac{1 - \frac{1}{s(s+5)}}{1 + \frac{10}{(s+1)(s+5)}} \right] \\
&= \lim_{s \to 0} \left[ \frac{1 - \frac{1}{s+5}}{1 + \frac{10}{(s+1)(s+5)}} \right] \\
&= \frac{1 - \frac{1}{5}}{1 + e^{-10/\ln 8}} \\
e_{ss} &= \frac{4}{15}
\end{align*}
\]
Steady State error due to Non UFB System:

=> The ss errors are calculate to only a closed loop stable UFB system.

=> If Non-UFB system is given it should be converted into UFB as follows:

\[
\begin{align*}
R(s) & \xrightarrow{+} X \xrightarrow{-} C(s) \\
H(s) & \xrightarrow{+} -X \\
& \xrightarrow{-} \rightarrow \\
& \downarrow \\
\end{align*}
\]

\[
C(s) = \frac{G_r}{1 + cH - G_r}
\]

\[
\text{Eqn. OLT of Non UFB System.}
\]
Find the $e_s$ to the given NUFB system to unit step input.

\[
\begin{align*}
C_{NF}(s) &= \frac{C_f}{1 + C_f H - C_f} \\
&= \frac{100}{s(s+10)} \\
&\quad + \frac{100}{s(s+5)(s+100)} - \frac{100}{s(s+10)} \\
C_{NF}(s) &= \frac{100(s+5)}{s(s+5)(s+100) + 100 - 100(s+5)} \\
&= \frac{100s + 500}{s^3 + 105s^2 + 500s + 100 - 100s + 500} \\
C_{NF}(s) &= \frac{100s + 500}{s^3 + 105s^2 + 400s - 400}
\end{align*}
\]

($Type = 0 = (T=0)$).
\[ e_{ss} = \frac{A}{1 + k} \]
\[ = \frac{1}{1 + \frac{500}{400}} = \frac{1}{1 - \frac{500}{400}} = -4 \]
\[ \therefore e_{ss} = -4 \]

**Method 2:**

\[ e_{ss} = \lim_{{s \to 0}} \frac{S \cdot R(s)}{1 + G(s)} \]
\[ = \lim_{{s \to 0}} \frac{8s \times \frac{1}{s}}{1 + \frac{100(s+5)}{s^3 + 105s^2 + 400s - 400}} \]
\[ = \frac{1}{1 - \frac{500}{400}} \]

\[ e_{ss} = \frac{4}{4 - 5} \]

\[ \Rightarrow e_{ss} = -4 \]