FUNDAMENTALS

CIRCUITS

- Current is intended to flow through all elements.
- This closed path concept is circuit.

[Diagram of a circuit with a 10V source and a 3Ω resistor]

NETWORKS

- Current does not necessarily flow through all the elements.

[Diagram of a parallel circuit with a 10V source and a 3Ω resistor]

- Circuits or networks are interconnection of various components to act together.
- Most of our practical systems are big interconnected networks, but we do circuit analysis to some of its parts where electrical energy flows.
- So circuits are building blocks of networks.

NETWORK COMPONENTS / ELEMENTS

All our applications are our components, but when these components are modelled as a circuit or network, we use fundamental components to model them like, V, I, R, L, C, etc.
Most of our practical components are non-linear in nature, but any non-linear sys. can be linearised for small incremental changes in time. But the same sys. under sudden big changes undergoes non-linear mode of operation.

We design all our new practically for specified...
ratings & as long as they obey Ohm's Law, Kirchoff's Law, Superposition Principle, etc. they are said to be linear.

- **BILATERAL ELEMENTS:**

  - \[ +\frac{4V}{2\Omega} \rightarrow -\frac{4V}{2\Omega} \rightarrow 2A \]
  - Properties independent of voltage → Polarity, Current → direction
  - \[ P_{loss} = 8W \]
  - \[ P_{loss} = 8W \]
  - \[ R \rightarrow \ell, L, C \]

- **UNILATERAL ELEMENTS:**

  - \[ +\frac{V}{-} \rightarrow -\frac{V}{+} \rightarrow \]
  - Forward biased Can conduct
  - Reverse biased Cannot conduct
  - \[ R_{ON} = 0 \Omega \]
  - \[ R_{OFF} = \infty \Omega \]
  - Properties depend on Voltage → Polarity, Current → direction
  - \[ \rightarrow BJT, FET, diode, etc. \]

Based on V-I characteristics:

- **Bilateral**
  - \[ I = III \]
- **Unilateral**
  - \[ I \neq III \]
• **Passive Elements** (sinks)
  - absorbs
  - dissipate
  - waste
  - convert
  - store

  \[ \text{eg:} \ R, L, C. \]

• **Active Elements** (sources)
  - energize
  - drive externally
  - deliver
  - give out

  \[ \text{eg:} \ V, I \]

Based on V-I characteristics:

\[ \begin{array}{c}
\text{V} + \downarrow \text{I} \\
\text{SOURCE} \\
\text{Sink} \\
\text{Source} \\
\text{Sink} \\
\text{Source} \\
\end{array} \]
1) The static V-I charac. of component is shown below, then component is

- Linear, active, bilateral
- Linear, passive, bilateral
- Non-linear, active, unilateral
- Non-linear, passive, bilateral

→ Non-linear
→ Unilateral
→ Both active & passive overall → Active

→ Non-linear
→ Active
→ Bilateral.

Active elements can act as passive elements, but passive elements cannot act as active.

E.g.: Capacitor always acts as a sink; either it charges or discharges.
**LUMPED PARAMETERS**

Properties:
- Simple
- Linear Algebraic equation
- Solutions are fast
- Approximated values.

**DISTRIBUTED PARAMETERS**

Properties:
- Complex
- Linear Differential Equations
- Solutions are Tediuous
- Very accurate values.
- Exact modelling
2) The relations like $V = nA$ holds good for

- (a) Lumped
- (b) Distributed
- (c) Lumped & Dist.
- (d) None

**NODE (n):**
A node is a point of interconnection or junction between 2 or more components.

**BRANCH (b):**
A branch is an elemental connection between two nodes.

**Degree of a Node (S):**
No. of branches incident or connected at any node represents its degree.
- If $S = 2$ → simple node (ns)
- $S > 2$ → principal node (np)

**NOTE:**
For any ckt or n/cw
\[ \sum_{i=1}^{n} S_i = 2 \times b \]

**MESH (m):**
Mesh is a closed path of ckt or n/cw which should not show further closed path in it.
• Loops (l):

Loops are all possible closed path of m+n.

\[ \text{m} = b - n + 1 \]

\[ m = b - n + 1 \]

\[ \text{Meshes are specifically called as independent loops}. \]

\[ \text{All meshes are by default loops but all loops are not meshes}. \]

\[ \text{In nodal analysis, we may neglect simple mode & one of the principal mode is considered as reference}. \]

\[ n = 5 \]
\[ b = 6 \]
\[ m = 2 \]
\[ l = 2 + 1 = 3 \]
\[ \Sigma S_i = 2 \times 2 + 2 + 3 + 3 = 12 \]
\[ b \times 2 = 6 \times 2 = 12 \]

\[ n = 89 \]
\[ b = 812 \]
\[ m = 4 \]
\[ l = l_1 + 6 = 19 \]
\[ \Sigma S_i = (7 \times 2) + 5 + 5 + 2 = 24 \]
\[ = 2 \times 12 \]
5. \[ n = 6 \]
   \[ b = 7 \]
   \[ m = 2 \]

6. \[ n = 5 \]
   \[ b = 6 \]
   \[ m = 2 \]

7. The minimum no. of eqns required to solve the set below is 4.

\[ m = b - n + 1 \]
\[ = 6 - 5 + 1 \]
\[ = 2 \]

Non-planar  \[ \rightarrow \]  Planar
**OHM'S LAW**

- L.T. I
- Temperature is constant.
- Uniform cross section of material.

\[ \text{J} = \frac{\text{V}}{\text{E}} \]  
(First Form)

Current density $\to$ Electric field intensity

density $\to$ Conductivity intensity

\[ \frac{I}{a} = \sigma \frac{V}{l} \]

\[ V = \left[ \frac{1}{\sigma a} \right] I \]

But $\frac{1}{\sigma} = \rho$ (resistivity)

\[ V = \left[ \frac{\rho l}{a} \right] I \]

\[ V = I R \]  
(Second Form)

Circuital form of Ohm's Law in 'R'

Graphically:
\[ V = (\frac{1}{R}) I \]

\[ V \]

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\[ L \rightarrow \text{Electromagnetic} \]

\[ \Psi = L i \quad (3^{\text{rd}} \text{ Form}) \]

\[ \Psi = N \Phi \]

\[ L \rightarrow \text{Flux linkage (Weber-T)} \]

\[ N \Phi = L i \]

\[ \frac{N \Phi}{dt} + \Phi \frac{dN}{dt} = L \frac{di}{dt} + i \frac{dL}{dt} \]

\[ V = L \frac{di}{dt} \quad (4^{\text{th}} \text{ Form}) \quad \text{Critical form of Ohm's law in 'L'} \]

\[ C \rightarrow \text{Electro-static} \]

\[ q = CV \quad (5^{\text{th}} \text{ Form}) \]

\[ \frac{dq}{dt} = C \frac{dV}{dt} + \left( \frac{V}{\frac{dc}{dt}} \right) \]

\[ i = C \frac{dV}{dt} \quad (6^{\text{th}} \text{ Form}) \quad \text{Critical form of Ohm's law in 'C'} \]

Graphically,

- \[ q \rightarrow V \]
  \[ \text{Slope} = C \]

- \[ \Psi \rightarrow i \]
  \[ \text{Slope} = L \]
* **DC Circuit Analysis**

- **Properties of DC Supply:**
  - Unipolar
  - Unidirectional
  - No change in phase/polarity
  - Power freq. = 0 Hz.

They are used in small, independent isolated power supply systems, where electrical energy can be stored in small capacities.

Eq:-
- Machine Tools
- Medical Instruments
- Cell phones, Toy
- Defence Applications

- **Standard DC Waveform:**

![DC Waveform Diagram]

- $V = 10V$
- $T \to \infty$
- $f \to 0$

![DC Circuit Diagram]
\* AC CIRCUIT ANALYSIS

- **Properties of AC supply:**
  - Bipolar
  - Bidirectional
  - Definite change in phase/polarity.
  - Power freq. exist (India = 50 Hz)

They are used in large, bulk, continuous power supply systems, where electrical energy cannot be stored.

Eq:
- Domestic
- Industrial Applications \} Robust
  \} Powerful

- **Standard AC Waveform:**

\[ V = V_m \sin(\omega t) \]

\[ \begin{align*}
\text{Sinusoidal} & \rightarrow \sin \\
\text{Cosine} & \rightarrow \cos
\end{align*} \]
VOLTAGE:

\[ V = \frac{dW}{dq} \]

→ It is the force (Emf), which can drive charges.

→ Units: volts \( \rightarrow \) T/C.

→ Range: kV, mV, V, MV, \( \mu \)V.

→ Symbols: \( V, V, V(t) \)

→ Circuit Symbols:

[Diagram showing AC and DC symbols with examples]

→ Examples:

DC → Cell, Battery, Fuel cells.
     P-V solar panels
     Rectified power sources
     Conventional DC converter

AC → UPS
     Inverter
     Alternator
CURRENT:

\[ i = \frac{dq}{dt} \quad \text{or} \quad I = \frac{\Delta Q}{\Delta t} = \frac{Q}{t} \]

→ It is rate of flow of charge
→ Units: ampere (A) or C/sec
→ Range: mA, mA, A, kA
→ Symbols: \( i, I, \bar{I}, i(t) \)

→ Circuit Symbols:

\[ \downarrow \quad \downarrow \]

\[ \text{DC} \quad \text{AC} \]

\[ \uparrow 10\,\text{A} \quad \uparrow 5\cos(100\,\text{t} + 32^\circ) \]

→ Examples:

DC → A DC series generator can be modeled as a DC current source.
   → A BJT can be modeled as a DC dependent current source.

AC → A Feeder can be modeled as an AC current source.

(Feeder → const. current density in conductors.)
**Resistance**:

- It is an electrical property of matter.
- "Resistor" is a component to model it.
  \[
  \frac{V}{I} = \frac{V}{I} = \frac{\text{V}}{\text{A}}
  \]
- Units: ohms (Ω), \( \frac{\text{V}}{\text{A}} \)

- **Range**: μΩ, mΩ, Ω, kΩ, MΩ, GΩ

\[ V = I \cdot R \Rightarrow I = \frac{V}{R} \]

**Basic Formula**:

\[ R = \frac{\mathcal{S} \cdot \ell}{a} \Omega \]

- \( l \): length of material
- \( a \): cross section area
- \( \mathcal{S} \): specific resistance (or) resistivity of material

- Resistance depends upon temperature.
  \[ R_t = R_0 [1 + \alpha (t - t)] \]

  - \( \alpha \): temperature co-eff. of resistance
  - \( \alpha \) is +ve → conductor
  - \( \alpha \) is -ve → semiconductor
Examples:

- All industrial & domestic wiring.
- Communication & Tx. lines.
- PCB components.

Eq.:

1) A cube shaped material has a resistance of $2 \Omega$ between any of its opposite faces. Now if this material is stretched in one direction by applying a linear force to double its original length, then the resistance between the two opposite stretched faces is ___.

\[ R_1 = \frac{s_1 l_1}{a_1} = 2 \]

\[ s_2 = s_1 \\
\ell_2 = 2 \ell_1 \\
v_1 = v_2 \\
\ell_1 a_1 = \ell_2 a_2 \\
\therefore a_1 = 2 a_2 \\
\therefore a_2 = \frac{a_1}{2} \]

\[ R_2 = \frac{s_2 \ell_2}{a_2} = \frac{s_1 (2\ell_1)}{a_1/2} = 4 \left( \frac{s_1 \ell_1}{a_1} \right) = 8 \Omega \]
**INDUCTANCE**

Electromagnetic property matter

→ "Inductor" is a component to model it.

\[ i(t) \quad \frac{L}{+} \quad V(t) \quad - \]

It is classified based on core material:

- Iron
- Ferrite
- Air

Units: Henry (H),

\[ \frac{V}{\text{sec}} \quad \frac{\text{A}}{\text{sec}} \]

Range: mH, nH, H

\[ V = L \frac{di}{dt} \quad \Rightarrow \quad i = \frac{1}{L} \int V \, dt \]

\[ i = \frac{1}{L} \int_{-\infty}^{t} V \, dt \quad \Rightarrow \quad i = \frac{1}{L} \int_{-\infty}^{t} V \, dt + \frac{1}{L} \int_{-\infty}^{0} V \, dt \quad \text{initial current} \]

\[ i = I_{0} + \frac{1}{L} \int_{0}^{t} V \, dt \]

**Basic Formula:**

\[ L = \frac{\mu N^{2} a}{l} \quad \text{H} \]

\[ \mu = \mu_{0} \mu_{r} \quad \text{permeability of core} \]

\[ \mu_{0} = 4\pi \times 10^{-7} \quad \text{H/m} \]

\[ \mu_{r} = 1 \quad \text{(air)} \]

\[ \mu_{r} > 1000 \quad \text{(iron)} \]
N → No. of turns of coil
a → cross-sectional area of core (m²)
l → effective length of magnetic flux path (m)

Examples:
→ Filter
→ Choke coils
→ Mic windings
→ Tx lines.

\[ l = 2\pi r \]

L = mean circumference.

CAPACITANCE:

→ Electrostatic property of matter

"Capacitor" is a component to model it.

\[ \frac{V(t)}{i(t)} = \frac{1}{C} \]

\[ i(t) = \frac{C}{t} \frac{dV}{dt} \]

\[ V(t) = \frac{1}{C} \int_{-\infty}^{t} i(t) \, dt + \frac{1}{C} \int_{-\infty}^{0} i(t) \, dt \]

 Vide: initial voltage

Units: Fenaday (F), \( \frac{A\cdot sec}{V} \)

Range: pF, nF, uF, mF

→ Electrolytic
→ Ceramic
→ Polyester

L → It is classified based on dielectric element/material.
\[ V = V_0 + \frac{1}{C} \int_{0}^{t} i \, dt \]

**Basic Formula**

\[ C = \frac{\varepsilon \cdot A}{d} \]

\[ \varepsilon = \varepsilon_0 \varepsilon_r \rightarrow \text{permittivity of dielectric} \]

\[ \varepsilon_0 = 8.85 \times 10^{-12} \, \text{F/m} \]

\[ \varepsilon_r = 1 \, \text{(air)} \]

\[ \varepsilon_r = 6 \, \text{(polyester)} \]

\[ \varepsilon_r = 760 \, \text{(ceramic)} \]

\[ d \rightarrow \text{dist. between electrodes (cm)} \]

\[ A \rightarrow \text{common cross-sectional area blow electrodes (m}^2) \]

**Examples**

- Filters
- Power system
- PDC circuits
- Tx chains \( \text{MF}/\text{HF}1 \, \text{km} \)

**Ideal Voltage Source**

\[ V(t) = \begin{cases} 10 \, \text{V} & \text{for } t > 0 \\ 0 & \text{for } t < 0 \end{cases} \]
Practical Voltage Source:

V-I Characteristics:

Ideal Current Source

Practical Current Source
V-I Characteristics:

\[ I \rightarrow \text{Ideal} \]
\[ I \rightarrow \text{Practical} \]
\[ 0 \rightarrow V \]

INDEPENDENT SOURCES:

- Properties
- Characteristics
- Values [Pres, Freq., etc.]

These are independent to any other parameter within or outside the circuit.

Examples:

- Ideal sources

\[ 5\, \text{V} \text{ 0} \]
\[ 10\, \text{A} \text{ 0} \]
\[ 10\sin t \text{ 0} \]
\[ 5\cos 2t \text{ 0} \]
DEPENDENT SOURCE:

- Properties
- Characteristics
- Values \([\text{Volts}, \text{Freq.}, \text{etc.}]\)

Examples:

- Practical sources
- BJT
- Solar cell

**Standard 4 Types:**

1. \(2V_x\)  
   - VDVS
   - VCVS

2. \(di_\beta\)  
   - CDVS
   - CCVS

3. \(e^{i\omega}\)  
   - CDVS
   - CCCS

4. \(V_x - V_y\)
   - VDVS
   - VCCS

Unlike independent sources, dependent sources cannot be suppressed in terms of resistance; as their models by themselves represent complex circuits.

**POWER:**

Rate of change in energy.

\[ P = \frac{dW}{dt} = \frac{dE}{dt} \]

Units: Watts or J/see

Range: mW, W, kW, MW, GW
1 hp = 746 W

\[ P_{\text{delivered}} = + Vi \ W = \frac{dW}{dq} \cdot \frac{dq}{dt} = \frac{dW}{dt} \]

\[ P_R = V_R i_R = i_R^2 R = \frac{V_R^2}{R} \]

\[ P_L = V_L i_L = L \frac{di}{dt} \ W \]

\[ P_C = i_C V_C = C V \frac{dV}{dt} \ W \]

ENERGY:

Capacity to do work (electrical)

\[ E = \int P \, dt \quad E = P \times t \]

Units: Joules (for W-sec)

1 unit of E.E. = 1 kWh

1 kWh = 1000 W \times 1 \text{ hr} = 500 W \times 2 \text{ hr} = 100 W \times 10 \text{ hr} = 2000 W \times \frac{1}{2} \text{ hr}
\[ \text{E}_{\text{delivered}} = +V \cdot i \cdot t \, \text{J} \]

\[ 1 \text{ kWh} = 36 \times 10^5 \text{J} \]

\[ E_R = \int \dot{P}_R \, dt = \int V_R \cdot i_R \, dt = \int i_R^2 \cdot R \, dt = \int \frac{V_0^2}{R} \, dt \]

\[ \rightarrow \text{But for } L,T,I \]

\[ E_R = V_R \cdot i_R \cdot t = i_R^2 \cdot R \cdot t = \frac{V_0^2}{R} \cdot t \, \text{J} \]

\[ E_L = \int \dot{P}_L \, dt = \int L \cdot \frac{di}{dt} \cdot dt \]

\[ \rightarrow \text{Now for } L,T,I \]

\[ E_L = \int L \cdot \frac{di}{dt} \cdot dt = \frac{1}{2} L \cdot i^2 \]

\[ \text{But } \psi = L \cdot i \]

\[ E_L = \frac{1}{2} L \cdot i^2 = \frac{1}{2} \psi \cdot i = \frac{\psi^2}{2L} \, \text{J} \]

\[ E_c = \int \dot{P}_c \, dt = \int C \cdot V \cdot \frac{dV}{dt} \cdot dt \]

\[ \rightarrow \text{But for } L,T,I \]

\[ E_c = \int C \cdot V \cdot \frac{dV}{dt} \cdot dt = \frac{1}{2} C \cdot V^2 \]

\[ \text{But } q = C \cdot V \]

\[ E_c = \frac{1}{2} C \cdot V^2 = \frac{1}{2} q \cdot V = \frac{q^2}{2C} \, \text{J} \]
ENERGY STORAGE CAPACITY in a Battery:

- Ahours - Hours (A hr)

**Example (1) Pencil Cell (AA) → 1.5 V, 500 mA hr**
- Cell + Plus → 3.7 V (1500 - 1800) mA hr
- Cell Batteries → 12 V, 40 A hr

**Example (2)**
- 40 A hr → 40 A × 1 hr
- → 20 A × 2 hr
- → 10 A × 4 hr
- → 80 A × \( \frac{1}{2} \) hr

KIRCHHOFF'S LAWS:

**1.** K 1st Law → KCL → Node

\[ \sum I_{\text{at node}} = 0 \]

\[ \frac{V}{R} \frac{1}{L} \text{s} + \frac{c}{d} \text{V} = I_S + I_R + I_L + I_C = 0 \]

**Example:**

\[ I_S = \frac{V}{R} + \frac{1}{L} \text{s} \text{V} + \frac{c}{d} \text{V} \]

2 + 22 + 7 + 1 = x + 6 + 9 + 10

\[ x = 12 \text{ mA} \]
(2) K2 Law $\rightarrow$ KVL $\rightarrow$ Mesh

$\sum V_{\text{mesh}} = 0$

$V_S = iR$

$V_L = L \frac{di}{dt}$

$V_C = \frac{1}{C} \int i dt$

$KVL$

$-V_S + V_R + V_L + V_C = 0$

$V_S = iR + L \frac{di}{dt} + \frac{1}{C} \int i dt$

$Final V_x = \frac{10 + 10 - 4}{2} = 6 V$

$V_x = 16 V$
Find \( E, V_1, V_2 \).

\[
\begin{align*}
0 - 4 - E - 8 &= 10 \\
E &= -22 \text{ V} \\
V_2 + 6 - 8 &= 10 \\
V_2 &= 12 \text{ V} \\
V_1 - 2 + 4 &= 0 \\
V_1 &= -2 \text{ V} \\
V_2 + 6 + E + 4 &= 0 \\
V_2 &= 12 \text{ V}
\end{align*}
\]

**Series Connection of Elements:**

\[
R_1 \quad R_2 \\
\frac{1}{R_s} = \frac{1}{R_1} + \frac{1}{R_2}
\]

\[
L_1 \quad L_2 \\
\frac{1}{L_s} = \frac{1}{L_1} + \frac{1}{L_2}
\]

\[
C_1 \quad C_2 \\
\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}
\]

Current sources of different values cannot exist in series. They violate KCL.

If 2 current sources are in series, they must be equal both in magnitude & direction.
Voltage sources of any value can be series.

**Parallel Connection of Elements:**

\[ R_p = \frac{1}{R_1} + \frac{1}{R_2} \]

\[ L_p = \frac{1}{L_1} + \frac{1}{L_2} \]

\[ C_p = C_1 + C_2 \]

- Never 2 ideal voltage sources of different values can exist in parallel. They violate KVL.
- If 2 voltage sources exist in parallel, they must be equal both in magnitude and polarity.
- Practical voltage sources can always exist in parallel.
- Current sources of any value can be in parallel.
Open Circuit / O.C.

- In an O.C., \( i = 0 \) for any voltage.

\[
R_{O.C.} = \frac{V}{i} = \infty \, \Omega
\]

- Any passive element in series to O.C. can be neglected.

Short Circuit / S.C.

- In a S.C., \( V = 0 \) for any (i).

\[
R_{S.C.} = \frac{0}{i} = 0 \, \Omega
\]

- Any passive element in parallel to S.C. can be neglected.

In high voltage engineering, O.C. live wire faults are more dangerous than S.C., to human beings.

However, S.C. always have protection, both at high & low level. VELTY leg designing some correct rated fuses.

- Resistance is offered by the path where current can flow as seen from target terminal.
1) $\text{Req.} = 1 + 3 = 4 \text{ S} \Omega$

2) $R_{xy} = 1 + 2 + 2 = 5 \text{ S} \Omega$

$\frac{3 \times 6}{9} = 2$

$R_{ab} = 1 + 6 + 2 = 9 \text{ S} \Omega$

$R_{xa} = 1 + 6 = 7 \text{ S} \Omega$

$R_{yb} = 1 + 2 = 3 \text{ S} \Omega$

3) $R_{xy} = \frac{1}{2} + 1 = \frac{3}{2} \text{ S} \Omega$

$R_{x2} = 0 \text{ S} \Omega$

$R_{yu} = 2 \text{ S} \Omega$

$R_{y2} = \frac{1}{2} + 1 = \frac{3}{2} \text{ S} \Omega$

$R_{yu} = \frac{1}{2} + 3 = \frac{7}{2} \text{ S} \Omega$

$R_{agy} = 3 + \frac{1}{2} = \frac{7}{2} \text{ S} \Omega$

$R_{aza} = 2 \text{ S} \Omega$
**CONDUCTANCE:**

- It is the ability to conduct electrically.
- It is used to further classify conductors (metals).

\[ G_i = \frac{1}{R} \quad \Rightarrow \quad G_i = \frac{a}{S l} \quad \Rightarrow \quad G_i = \frac{\sigma a}{l} \]

**Units:** mho (Ω) \( \rightarrow \) Siemens (S) = \( \frac{A}{V} \)

\[ G_i = \frac{1}{R} \quad \Rightarrow \quad G_i = \frac{a}{S l} \quad \Rightarrow \quad G_i = \frac{\sigma a}{l} \]

\[ \Rightarrow \quad \frac{1}{S} = \sigma \quad \text{(conductivity)} \]

**Units for** \( \sigma \) \( \rightarrow \) (Siemens \cdot m\(^{-1}\))
\( \Rightarrow \) S/m (or) Siemens/m

\[ V = \frac{I}{G_i} \quad \Rightarrow \quad I = V \cdot G_i \]

\[ P_{G_i} = V_{G_i} \cdot I_{G_i} = \frac{I_{G_i}^2}{G_i} = \frac{V_{G_i}^2}{G_i} \cdot G_i \quad \text{W} \]

\[ \frac{1}{G_{eq}} = \frac{1}{G_{i1}} + \frac{1}{G_{i2}} \]

\[ G_{eq} = G_{i1} + G_{i2} \]

Based on Conductivity:

- **Rank 1:** Silver
- **Rank 2:** Copper (used in high current density compact systems)

\( \rightarrow \) Domestic, Industrial, M/C, PCB
3. Gold
4. Aluminium — used in external Tx & distribution lines
   • cheap
   • lightweight

Voltage Division Rule

Series connected elements only

\[ V_{L_1} = V \left( \frac{L_1}{L_1 + L_2} \right) \]
\[ V_{L_2} = V \left( \frac{L_2}{L_1 + L_2} \right) \]

\[ V_{C_1} = V \left( \frac{C_2}{C_1 + C_2} \right) \]
\[ V_{C_2} = V \left( \frac{C_1}{C_1 + C_2} \right) \]

\[ V_{G_{11}} = V \left( \frac{G_{12}}{G_{11} + G_{12}} \right) \]
\[ V_{G_{12}} = V \left( \frac{G_{11}}{G_{11} + G_{12}} \right) \]
\[ V_{R_3} = V \left[ \frac{R_3}{\sum_{i=1}^{n} R_i} \right] \]

But suppose these are capacitors:
\[ V_{C_3} = V \left[ \frac{1/C_3}{\sum_{i=1}^{n} 1/C_i} \right] \]

**Current Division Rule:**

Parallel connected elements only.

\[ I_{R_1} = I \left[ \frac{R_2}{R_1 + R_2} \right] \]
\[ I_{R_2} = I \left[ \frac{R_1}{R_1 + R_2} \right] \]

\[ I_{L_1} = I \left[ \frac{L_2}{L_1 + L_2} \right] ; \quad I_{L_2} = I \left[ \frac{L_1}{L_1 + L_2} \right] \]

\[ I_{C_1} = I \left[ \frac{C_2}{C_1 + C_2} \right] ; \quad I_{C_2} = I \left[ \frac{C_1}{C_1 + C_2} \right] \]

\[ I_{G_1} = I \left[ \frac{G_2}{G_1 + G_2} \right] ; \quad I_{G_2} = I \left[ \frac{G_1}{G_1 + G_2} \right] \]
\[ I_{R_3} = I \left[ \frac{R_1 R_2}{R_1 R_2 + R_2 R_3 + R_3 R_4} \right] \]

\[ I_{c_3} = I \left[ \frac{1/C_1 \cdot 1/C_2}{1/C_1 + 1/C_2 + 1/C_3 + 1/C_4} \right] \]

\[ I_{R_2} = I \left[ \frac{R_1 R_2 R_3 R_4}{R_1 R_2 R_3 + R_2 R_3 R_4 + R_3 R_4 R_1 + R_4 R_1 R_2} \right] \]
DELTA-TO-STAR

\[ R_1 = \frac{R_a R_c}{R_a + R_b + R_c} \]

\[ R_2 = \frac{R_a R_b}{R_a + R_b + R_c} \]

\[ R_3 = \frac{R_c R_b}{R_a + R_b + R_c} \]

\[ L_1 = \frac{L_a L_c}{L_a + L_b + L_c} \]

\[ L_2 = \frac{L_a L_b}{L_a + L_b + L_c} \]

\[ C_1 = \frac{1}{C_a} \cdot \frac{1}{C_c} = \frac{1}{C_a} + \frac{1}{C_b} + \frac{1}{C_c} \]

\[ \frac{1}{C_2} = \frac{1}{C_a} \cdot \frac{1}{C_b} = \frac{1}{C_a} + \frac{1}{C_b} + \frac{1}{C_c} \]

\[ \frac{1}{G_{11}} = \frac{1}{G_{a}} \cdot \frac{1}{G_{c}} \cdot \frac{1}{G_{a} + \frac{1}{G_{b}} + \frac{1}{G_{c}}} \]
\[
\frac{1}{G_2} = \frac{1}{G_a} \cdot \frac{1}{G_b} = \frac{1}{G_a} + \frac{1}{G_b} + \frac{1}{G_c}
\]

**STAR - TO - DELTA**

\[R\]
\[
R_a = R_1 + R_2 + \frac{R_1 R_2}{R_3}
\]
\[
R_b = R_2 + R_3 + \frac{R_2 R_3}{R_1}
\]
\[
R_c = R_1 + R_3 + \frac{R_1 R_3}{R_2}
\]

\[L\]
\[
L_a = L_1 + L_2 + \frac{L_1 L_2}{L_3}
\]

\[C\]
\[
\frac{1}{C_a} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}
\]

\[G\]
\[
\frac{1}{G_{1a}} = \frac{1}{G_1} + \frac{1}{G_{12}} + \frac{1}{G_3}
\]
SOURCE TRANSFORMATION

An ideal voltage source in series with resistors can be converted into an ideal current source in parallel with the same resistors, across the same terminals, and vice versa.

\[ V \rightarrow I \quad I = \frac{V}{R} \]

\[ I \rightarrow V \quad V_x = I_x R \]

Ratings / Specifications:

They represent the maximum permissible or allowable safe values for continuous operation of an electrical device.

Most of our electrical or electronic components or devices will have voltage, current, power, frequency ratings etc.

\[ V, I \rightarrow 2V, 2I \]
\[ I \uparrow \rightarrow \text{conductor cross-sectional area} \downarrow \\\nV \uparrow \rightarrow \text{insulation withstanding capacity} \uparrow \\\n\rightarrow \text{Most of our Electrical & Electronic equipment are designed for constant Voltage.} \\\n\rightarrow \text{But their current carrying capacity depend upon loading level.} \]

<table>
<thead>
<tr>
<th>Low Wattage</th>
<th>High Wattage</th>
</tr>
</thead>
<tbody>
<tr>
<td>V \uparrow</td>
<td>V \downarrow</td>
</tr>
<tr>
<td>I \downarrow</td>
<td>I \uparrow</td>
</tr>
<tr>
<td>A \downarrow</td>
<td>A \uparrow</td>
</tr>
<tr>
<td>R \uparrow</td>
<td>R \downarrow</td>
</tr>
</tbody>
</table>

\[
\text{which glows brighter?} \quad \begin{align*}
40W \\
60W
\end{align*}
\]

\[
\text{which glows brighter?} \quad \begin{align*}
40W \\
60W
\end{align*}
\]

\[
\text{If load } \uparrow \Rightarrow \text{most power is drawn} \\
\Rightarrow \text{more current is drawn as volts remain same}
\]

\[
\rightarrow \text{The load always decides the power drawing capability or current capability of the system.}
\]
**Resistor Reduction Technique:**

1. \( \frac{4 \times 8}{12} = \frac{8}{3} \ \Omega \)

2. \( 1 + \left[ \frac{2}{1} \right] = \frac{10}{5} \ \Omega \)

3. \( \frac{3 \times 7}{10} = \frac{21}{10} \)

4. \( \text{same as} \ \ \frac{a}{b} \ \ \text{at 2nd.} \)
Open circuit the current source

\[ R_{xy} = \frac{3 \times 10^6}{2} = 1.5 \times 10^6 \Omega \]

Node shifting tech.

\[ R_{xy} = \frac{2}{11} \times 2 = 1 \]

\[ 7 \Omega \]

\[ 2 \Omega \]
\[ R_{xy} = \frac{1 + 2}{11} \parallel \frac{3 + u}{10} \]
\[ = \frac{3 \times 7}{10} \]
\[ = \frac{21}{10} \]

\[ R_{xy} = \frac{1 + 2}{11} \parallel \frac{3 + u}{10} \]
\[ = \frac{3 \times 7}{10} \]
\[ = \frac{21}{10} \]

\[ R_{x} = \frac{1 + 2}{11} \parallel \frac{3 + u}{10} \]
\[ = \frac{3 \times 7}{10} \]
\[ = \frac{21}{10} \]

\[ R_{x} = \frac{1 + 2}{11} \parallel \frac{3 + u}{10} \]
\[ = \frac{3 \times 7}{10} \]
\[ = \frac{21}{10} \]

\[ = \frac{21}{10} \]
\[ R_{y6} = \frac{4 \times 1(1+3+4)}{10} = \frac{12}{5} \]

\[ R_{x5} = \frac{3(1)(1+2+4)}{1} = 3 \]

\[ R_{y5} = \frac{2}{1}(1+3+4) \]

\[ R_{xy} = 8 + \left(2 \frac{1}{1} \frac{3}{1} \frac{1}{6} \right) = 8 + 1 = 9 \]

\[ R_{bc} = \frac{3 \frac{1}{1} \frac{6}{1} \frac{1}{2} \frac{1}{1}}{1} = 1 \]

Always convert internal star to delta.

Converting delta to star centre point may not be same.

\[ R_{ao} = 7 \frac{9}{3} R_{bc} = 1 \]
\[ I = \frac{10}{2} = 5 \text{ A} \]

\[ \Rightarrow 2/3 \text{ R} \]

\[ R_{xy} = 2/12 = 1 \sqrt{2} \]

\[ R_{xy} = 2/16 = \frac{3}{2} \sqrt{2} \]
21. \[ R_{xy} = 1 + 5 + 4 + 5 + 6 + 7 = 28 \Omega \]

22. \[ R_{eq} = 50 \text{ Ohm} \]

\[ R = \frac{30 + [60/11(14 + R)]}{50} \]

\[ \frac{60(14 + R)}{R + 79} = 20 \]

\[ 412 + 3R = R + 7 \frac{5}{3} \]

\[ R = 16 \Omega \]

23. The V-I relation in a component is \[ V = i^2 \]

\[ R_{dynam} = R_{static} \]

\[ R_{static} = \frac{V}{I} \]
\[ V = i^2 \]
\[ \frac{dv}{dt} = 2 \frac{di}{dt} \]

\[ R_{dynamic} = \frac{2}{R_{static}} \]

\[ R = 4 + \left[ \frac{211R}{2} \right] = 4 + \frac{2R}{2+R} \]
\[ = \frac{4R + 2R}{2+R} = \frac{6R}{2+R} \]
\[ 2R + R^2 = 8 + 2R + 4R \]
\[ R^2 - 4R - 8 = 0 \]
\[ R = \frac{4 \pm \sqrt{16 - 4(-8)}}{2} = \frac{4 \pm \sqrt{48}}{2} = \frac{4 \pm 2\sqrt{3}}{2} \]
\[ R = (2 \pm 2\sqrt{3}) \Omega \]

\[ \text{or} \quad R = (2 + 2\sqrt{3}) \Omega \]

\[ C = \frac{1}{1 + C} \]
\[ C = \frac{1}{2 + C} \]
\[ C = \frac{1 + C}{2 + C} \]
\[ C = \frac{-1 \pm \sqrt{5}}{2} \]
\[ C = \frac{\sqrt{5} - 1}{2} \text{ F} \]
Applying KVL in the above loop:

\[-V + \frac{I}{3} (1) + \frac{I}{6} (1) + \frac{I}{3} (1) = 0\]

\[V = I \left[ \frac{1}{3} + \frac{1}{6} + \frac{1}{3} \right] = I \left[ \frac{5}{6} \right]\]

\[R_{AB} = \frac{V}{I} = \frac{5}{6} \text{ \Omega}\]

\[R_{AB} = \frac{5}{6} \Omega\]

\[L_{AB} = \frac{5}{6} \text{ L}\]

\[C_{AB} = \frac{6}{5} \text{ F}\]

\[G_{AB} = \frac{6}{5} \text{ S}\]

\[\text{KVL for capacitors:}\]

\[-V + \frac{1}{C} \int_{0}^{t} \frac{I}{3} dt + \frac{1}{C} \int_{0}^{t} \frac{I}{6} dt + \frac{1}{C} \int_{0}^{t} \frac{I}{3} dt = 0\]

\[\therefore V = \frac{1}{C} \left[ \frac{1}{3} + \frac{1}{6} + \frac{1}{3} \right] \int_{0}^{t} I dt\]

\[= \frac{1}{C} \left[ \frac{5}{6} \right] \int_{0}^{t} I dt\]

\[\frac{1}{C_{AB}} = \frac{1}{C} \left[ \frac{5}{6} \right]\]

\[\therefore C_{AB} = \frac{6}{5}\]
Use Superposition principle.

Step 1

Step 2

Each \( r = 1.5 \)
\[ KVL \quad -1 + i_T + 2i_T + \frac{1}{2} + 3i_T = 0 \]

\[ 6i_T = \frac{1}{2} \quad \Rightarrow \quad i_T = \frac{1}{12} \]

\[ R_{ao6} = \frac{1}{i_T} = \frac{1}{\frac{1}{12}} = 12 \Omega \]
\[ R_{xy} = \frac{1}{i_T} \]

\[ i_T = \frac{1}{100} + \frac{1}{10k} + \frac{99}{10k} \]

\[ = \frac{1}{100} + \frac{1}{10k} \left[ \frac{100}{100} \right] \]

\[ = \frac{1}{100} + \frac{1}{100} = \frac{2}{100} = \frac{1}{50} \]

\[ R_{xy} = 50 \, \Omega \]

**Effective resistance is the resistance offered by the circuit under working condition.**

\[ R_{eff} = \frac{5}{i_T} = \frac{5}{10} = 0.5 \, \Omega \]
What is $R_{eff}$ seen by the voltage source?

\[ KVL \]
\[-V + \frac{3i}{4} \times 4 = 0\]
\[ \therefore V = 3i \]

$R_{eff} = \frac{V}{i} = 3 \, \Omega$

12 mA
11 mA
10 mA
1 mA

7 A
2 A
10 A
2 A
12 A
6 A

50 V
+ 5 V
+ 20 V
+ $V_2$
+ $V_3$
+ 10 V

10 - 25 = $V_1$
$V_1 = -15 \, V$
$V_1 = 30 \, V$
$V_3 = -4.5 \, V$

10 - 20 = 0
$V_2 = -30 \, V$

$10 + 2V_1 - 30 = 0$
$2V_1 = 40$
$V_1 = 20$
\[ I_A = \frac{20}{5} = 4 \, \text{A} \]

\[ V_{2c} = 25 - 4I_A = 25 - 16 \]
\[ I = \frac{20}{10} = 2 \, \text{A} \]

\[ V_2 = -30 + 5I + 40 + V_1 \]
\[ \therefore V_2 = V_1 + 20 \]

\[ i = \frac{3/2 \times 11}{3/2 + 2} = \frac{33}{11} = 3 \, \text{A} \]

\[ V = \frac{1 \times 20}{4 + 1} = \frac{20}{5} = 4 \, \text{V} \]

\[ i_1 = \frac{10}{2} = 5 \, \text{A} \]
\[ i_2 = 0 \]
\[ V_2 = 0 \]

\[ i = \frac{9 \times 4}{4 + 2} = \frac{36}{6} = 6 \, \text{A} \]

\[ S \rightarrow \text{Siemens (i.e., conductance)} \]
1 = \frac{1}{3} \times 3 \Rightarrow I = 3 \, A

V_5 = 3 \times (10 + \frac{2}{3}) = 32 \, V

i = \frac{-32}{8} = -4 \, A

\frac{v_0}{4} = 4 \, V, \quad R = \frac{9}{12} \, \Omega

I = \frac{16}{16} = 1 \, A

I = 2 \times 28 = \frac{2}{2 + \frac{6}{11}}

-1_x = 2 \times 28 \times \frac{4}{4(2) + 3} = 6

\frac{10 - 3i_0}{2} = i_0 \Rightarrow 5 \times 0 = 0

i_0 = 2 \, A

P = -6 \times 2 = -12 \, W

\frac{2s_{0}}{2} \, \frac{v_0}{2} = 0

\therefore V_0 = \frac{10}{2} = 5 \, V

i_b = \frac{2.8 \times 15}{5} = 10 \, \text{mA}

V_{sc} = -1000 (0.5) = -50 \, V
\[ \frac{V_{2x}}{2} + 10 + 2V_{2x} = 0 \]
\[ 5V_{2x} = -20 \]
\[ V_{2x} = -4 \text{ V} \]

Power delivered by dependent source:
\[ V_0 = -\left( \frac{3 \times 10^2}{3 + 2} \right) = -6 \text{ V} \]
\[ P = +10 \times 12 = 120 \text{ W} \]

\[ V_0 = \frac{30 \times x^2}{3} = 20 \text{ V} \]
\[ i_o = \frac{4}{3} = 1 \text{ A} \]
\[ I = \frac{3}{q} = \frac{1}{3} \]
\[ V_0 = 6 \times \frac{1}{3} = 2 \text{ V} \]

\[ I = \frac{4}{v} = 1 \text{ A} \]
\[ i_o = \frac{6}{q} = \frac{2}{3} \text{ A} \]
\[ V_0 = -12 \left( \frac{2}{3} \right) + 6 \left( \frac{1}{3} \right) = -6 \text{ V} \]

\[ \frac{V}{q} = \frac{2}{2} \]
\[ \text{Data insufficient} \]

\[ \text{We missed the value across the current source} \]
\[ V_{x} = 28 \text{V} \]

\[ V_{a} - V_{b} = 6 \text{V}, \quad V_{c} - V_{d} = 7 \text{V} \]

**Find all branch currents**

\[ 2(1-x) - 3y - x = 0 \]
\[ 3x + 3y = 2 \]  \[ \text{Eq. 1} \]

\[ 3y + 5(1-x+y) - 4(x-y) = 0 \]
\[ 4x - 12y = 5 \]  \[ \text{Eq. 2} \]

From Eq. 1 and Eq. 2:

\[ x = 0.619, \quad y = 0.047 \]
Methods of Analysis:

Mesh Analysis = KVL + Ohms' Law

$[I \uparrow \downarrow V \downarrow \uparrow]$  

Nodal Analysis = KCL + Ohms' Law

$[V \uparrow I \downarrow \downarrow]$  

In nodal analysis, we can eliminate the use of simple nodes, if not required.

Mesh Analysis

1. $10 - 3i_1 + 2i_2 = 0$
   $3i_1 - 2i_2 = 10$

2. $5 - 5i_2 + 2i_1 = 0$
   $\Rightarrow 2i_1 - 5i_2 = -5$

From 1 & 2:

$i_1 = \frac{60}{11}$  

$i_2 = \frac{35}{11}$

Now:

$i = i_1 - i_2$

$i = \frac{25}{11}$ A
Nodal Analysis

\[ \frac{V_1}{1} - \frac{V_1}{2} + \frac{V_1}{3} = 0 \]
\[ \Rightarrow 6V_1 - 60 + 3V_1 + 2V_1 - 10 = 0 \]
\[ V_1 = \frac{50}{11} \text{ V} \]

Now,
\[ i = \frac{V_1}{2} = \frac{25}{11} \text{ A.} \]

Find power delivered by current source using mesh & nodal.

\[ -10 + i - 2 + 2i = 0 \]
\[ i = 4 \text{ A} \]
\[ P_{\text{deliv.}} = + 2 \times 14 = 28 \text{ W.} \]

\[-10 + i_1 + 2(i_1 - i_2) = 0 \]
\[ 3i_1 - 2i_2 = 10 \quad (1) \]

By comparison: \[ i_2 = -2 \quad (2) \]
\[ P_{\text{del}} = 28 \text{ W.} \]
\[ 3i_1 = 6 \Rightarrow i_1 = 2 \]

\[ \frac{V_1}{1} - \frac{V_1}{2} - 2 = 0 \]
\[ \Rightarrow V_1 = 8 \text{ V.} \]
\[ P_{\text{del}} = 28 \text{ W} \]

Find \( i \) using mesh & nodal analysis.

\[ -10 - i - 2 + 5i = 0 \]
\[ i = 2 \]
Here $i_2 = i$
\[10 - i_1 - 5i = 0\]
\[i_1 + 5i = 10\]
\[i_1 = i = 2\]
\[i_2 = 2\]

\[6i = 12 \Rightarrow i = 2\ A\]

\[\frac{V_1 - 10}{1} + 2 + \frac{V_1}{5} = 0\]
\[6V_1 = 60 \Rightarrow V_1 = 10\ V\]
\[i = \frac{V_1}{5} = \frac{10}{5} = 2\ A\]

Find the power delivered by 5V source using mesh and nodal analysis.

\[P_{del} = 5\ W\]

\[i_2 + i_3 = 0\]
\[2i_2 = -2\]
\[i_2 = -1\]

\[10 + 2i_1 + 2(i_2 - i_3) = 0\]
\[2i_1 - i_2 = 5\]
\[-2i_1 + 2i_2 + i_3 = 5\]

\[i_2 + i_3 = 2\]

\[P_{del} = 5\ W\]

\[V_1 - 10 + \frac{V_1}{2} - 2 + \frac{V_2}{2} = 0\]
\[V_1 + V_2 = 7\]
\[V_1 - V_2 = 5\]

\[V_1 = 6\ V\]
\[V_2 = 1\ V\]

\[P_{deliv.} = 5 \times 1\]
\[= 5\ W\]
\[ V = \frac{1}{8} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -6 \\ 6 \end{bmatrix} \]

NoDAl:

\[ \begin{aligned} \frac{V_3 - V_2}{V_2} & = \frac{V_2 - V_3}{V_3} = 10 \\ \frac{V_2 - 10}{V_2} + \frac{V_3 - V_3}{V_3} & = 0 \end{aligned} \]

\[ \begin{aligned} \frac{3V_2}{V_2} & = 10 \\ V_2 & = 10 \end{aligned} \]

\[ \begin{aligned} V_3 & = \frac{V_2}{10} + \frac{V_3}{V_3} = 10 \\ V_3 & = \frac{V_3}{V_3} + \frac{V_3}{V_3} = 0 \end{aligned} \]

\[ \begin{aligned} V & = 10V_1 \\ V & = 0 \end{aligned} \]

\[ V = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \\ 0 \end{bmatrix} \]

\[ \begin{align*} & V_1 = 8 \\ & V_2 = 0 \\ & V_3 = 0 \end{align*} \]

\[ \text{Find } V \text{ using nodal analysis.} \]
Find \( i \) using mesh & nodal methods.

**Mesh:**

\[ -i_1 \cdot i_3 - i_1 = 2 \quad (1) \]
\[ 3i_2 - i_1 - i_3 = 0 \quad (2) \]
\[ 10 - i_1 - 3i_3 + 2i_2 = 0 \quad (3) \]

Now, from (2) & (1):
\[ 3i_2 - i_3 + 2 - i_3 = 0 \]
\[ 3i_2 - 2i_3 = -2 \quad (4) \]

Now, from (1) & (3):
\[ -i_3 + 2 - 3i_3 + 2i_2 = 0 \]
\[ 2i_2 - 4i_3 = 10 \quad (5) \]

\[ 6i_2 - 4i_3 = -4 \]
\[ 6i_2 - 12i_3 = -24 \]
\[ 8i_3 = -28 \]
\[ i_3 = -\frac{7}{2} \]

**Nodal:**

\[ V_1 = 10 \text{ V} \quad (6) \]
\[ \frac{V_2 - 10}{1} + \frac{V_2 - V_3}{1} - 2 = 0 \]
\[ 2V_2 - V_3 = 12 \quad (7) \]
\[ \frac{V_3 - 10}{1} + \frac{V_3 - V_2}{1} + \frac{V_3}{2} = 0 \]
\[ -2V_2 + 3V_3 = 10 \quad (8) \]
\[ 4V_3 = 32 \]
\[ V_3 = 8 \text{ V} \quad \Rightarrow \quad i = \frac{8}{\frac{7}{2}} = \frac{16}{7} \text{ A} \]
What is the power delivered by the voltage source?

Mesh:

\[ i_2 - i_1 = 10 \]  \[ \ldots (0) \]

\[ 3i_1 + 2i_2 - 2i_3 = -5 \]
\[ 3(i_2 - 10) + 2i_2 - 2i_3 = -5 \]
\[ 3i_2 - 30 + 2i_2 - 2i_3 = -5 \]
\[ \therefore 2i_2 - 2i_3 = 25 \]  \[ \ldots (2) \]

\[ 3i_3 - 2i_1 = 5 \]
\[ 3i_3 - 2(i_2 - 10) = 5 \]
\[ \therefore -2i_2 + 3i_3 = -15 \]  \[ \ldots (3) \]

\[ i_3 = \frac{4(\frac{45}{8}) - 25}{4} \]
\[ = \frac{45 - 50}{4} = -\frac{5}{4} \]

\[ P_{\text{del.}} = -5 \times \left( \frac{45 + 5}{8} \right) \]
\[ = -\frac{275}{8} \text{ W} \]

Nodal:

\[ \frac{V_1}{1} + \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{1} = 0 \]
\[ 5V_1 - V_2 - 2V_3 = 0 \]  \[ \ldots (1) \]

\[ (V_2 - V_3) = 5 \]  \[ \ldots (2) \]
\[
\frac{V_2 - V_1}{2} - 10 + \frac{V_3}{1} + \frac{V_3 - V_1}{1} = 0
\]
\[
-3V_1 + V_2 + 4V_3 = 20 \quad \text{(3)}
\]
From (1) and (3):
\[
7V_1 - V_2 = 20
\]
From (2) and (3):
\[
5V_1 - 3V_2 = -10
\]
\[
16V_1 = 70
\]
\[
V_2 = 7(\frac{35}{8}) - 20
\]
\[
V_2 = \frac{85}{8}
\]
\[
V_1 = \frac{35}{8}
\]
\[
I = \frac{35}{8}
\]
\[
5V
\]
\[
10 - \frac{25}{8}
\]
\[
- \frac{25}{8}
\]
\[
\text{Resultant Current: } I = \frac{35}{8}
\]
\[
\text{Resultant Voltage: } V = \frac{85}{8}
\]
\[
\text{Power delivered by the source: } P_{\text{deliv}} = -5\left(10 - \frac{25}{8}\right)
\]
\[
\text{Power delivered by the source: } P_{\text{deliv}} = -\frac{275}{8} \text{ W}
\]

**Mesh:**

\[
i_1 + i_2 + i_3 = 10 \quad \text{(1)}
\]
\[
i_2 - i_1 = 3 \quad \text{(2)}
\]
\[
i_2 = 3 + i_1
\]
\[
i_2 - i_3 = 4 \quad \text{(3)}
\]
\[
3 + i_1 - i_3 = 4
\]
\[
i_3 = i_1 - 1
\]

Find power delivered by each source using mesh and nodal methods.
1. \( i_1 + 3i_1 + i_1 - 1 = 10 \)
2. \( 3i_1 = 8 \Rightarrow i_1 = \frac{8}{3} \)
3. \( P_{\text{deliv.}} = 10 \times \frac{8}{3} = \frac{80}{3} \text{ W} \)

\[ \text{NODAL:} \]

\[ \frac{V_1 - 10}{1} + \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{1} = 0 \]

\[ 5V_1 - V_2 - 2V_3 = 20 \] \[ \text{--- (1)} \]

\[ \frac{V_2 - V_1}{2} + 4 - 3 = 0 \]

\[ V_2 - V_1 = -2 \] \[ \text{--- (2)} \]

\[ \frac{V_3}{1} - 4 + \frac{V_3 - V_1}{1} = 0 \]

\[ -V_1 + 2V_3 = 4 \] \[ \text{--- (3)} \]

From (2): \( 5V_1 - V_2 - 2 - 2V_3 = 20 \)

\[ V_3 = 4 + \frac{22}{3} \]

\[ \frac{22}{3} \]

\[ V_3 = \frac{34}{6} \]

\[ \text{Now,} \]

\[ P_{\text{deliv.}} = 10 \times \frac{8}{3} = \frac{80}{3} \text{ W} \]

\[ I = \frac{10 - \frac{22}{3}}{1} = \frac{8}{3} \text{ A} \]
\[ i - 10 + i - 1 + i_3 = 0 \]
\[ 3i = 8 \]
\[ i = \frac{8}{3} \]

\[ P_{\text{deliv.}} = 10 \times \frac{8}{3} = \frac{80}{3} \text{ W} \]

Find the power delivered by the current source, using mesh 8 model.

**Mesh**:

\[ i_1 + i_2 + 2i_3 = 0 \] \[ 1 \times \]

\[ 2i_3 = 15 \Rightarrow i_3 = \frac{15}{2} \]

\[ i_1 + i_2 = -15 \]

\[ -i_1 + i_2 = 2 \]

\[ i_2 = -\frac{13}{2} \]

\[ i_1 = -\frac{17}{2} \]

\[ P_{\text{deliv.}} = -2 \times \frac{3}{2} = -3 \text{ W} \]
**Nodal**

\[
\frac{V_1}{1} + \frac{V_1 - V_3}{2} + \frac{V_3}{1} + \frac{V_3 - V_1}{2} = 0
\]

\[
3V_1 - V_3 = 0
\]

\[
V_1 + V_3 = 2 \quad \text{(1)}
\]

\[
V_1 - V_2 = 10 \quad \text{(2)}
\]

\[
V_2 - V_3 = 5 \quad \text{(3)}
\]

\[
\Rightarrow V_2 + V_1 = 7.
\]

\[
2V_1 = 17
\]

\[
V_1 = \frac{17}{2}
\]

\[
V_2 = \frac{17}{2} - 10 = \frac{-13}{2}
\]

\[
V_2 = \frac{-13}{2}
\]

\[
\text{\(P_{\text{deliv}}\)} = 2 \times \left(-\frac{3}{2}\right) = -3 \text{\(W\)}.
\]

\[
\frac{V - 10 + V + 5}{1} = 0
\]

\[
2V = 3
\]

\[
V = \frac{3}{2} V
\]

\[
\Rightarrow P_{\text{deliv}} = -\frac{3}{2} \times 2 = -3 \text{\(W\)}
\]

\[
\text{\(10\text{\(\Omega\)}
\]
MESH:

\[ i_1 - i_2 = 4 \]  \hspace{1cm}  \[ i_3 = 9 \text{ A} \]  \hspace{1cm}  \[ i_3 - i_2 = 7 \]  \implies  \[ i_2 = 2 \text{ A} \]

\[ i_1 = 6 \text{ A} \]

NODAL:

\[ \frac{V_1 - 10}{1} + \frac{V_2 - V_1}{2} + q = 0 \]  \hspace{1cm}  \[ V_2 - V_1 \]  \hspace{1cm}  \[ + q_1 - 7 = 0 \]
\[ \frac{V_3}{1} + q_2 \cdot 7 - q = 0 \]

Now,

\[ P_{\text{delivered}} = 10 \times 6 = \boxed{60 \text{ W}} \]

What is the power delivered by the source using mesh & nodal?

MESH:

\[ i_2 - i_1 = 4 \]  \hspace{1cm}  \[ 3i_3 - i_1 - i_2 = -15 \]  \implies  \[ i_1 + i_2 = 3i_3 + 15 \]

\[ i_1 + i_2 + 2i_3 = -15 \]  \implies  \[ 3i_3 + 15 + 2i_3 = -15 \]

\[ P_{\text{delivered}} = 15 \times \frac{15}{2} = 112.5 \text{ W} \]

\[ i_3 = \boxed{5 \text{ A}} \]
NODAL:

\[
\frac{V_1}{1} + \frac{V_1 - V_2}{1} + \frac{V_1 - 15 - V_3}{2} = 0
\]

\(\therefore 5V_1 - 2V_1 - V_3 = 15 \quad (1)\)

\[
\frac{V_2 - V_1}{1} + \frac{V_2 - V_3}{1} - 10 = 0
\]

\(\therefore -V_1 + 2V_2 - V_3 = 10 \quad (2)\)

\[
\frac{V_3}{1} + \frac{V_3 - V_2}{1} + \frac{V_3 + 15 - V_2}{2} = 0
\]

\(\therefore -V_1 - 2V_2 + 5V_3 = -15 \quad (3)\)

From (1) and (3):

\(4V_1 - 2V_3 = 25\)

From (2) and (3):

\(-2V_1 + \frac{4V_3}{8} = -\frac{5}{10}\)

\(6V_3 = 15\)

\[V_3 = \frac{15}{6} = \frac{5}{2} V\]

\[V_1 = \frac{25 + \frac{15}{3}}{4} = \frac{90}{12} = \frac{15}{2} V\]

\[I = \frac{V_3 + 15 - V_1}{2} = \frac{\frac{5}{2} + 15 - \frac{15}{2}}{2} = \frac{20}{4} = 5 A\]

\[P_{\text{delivered}} = 15 \times 5 = 75 W\]
Find $V_\alpha$ using mesh & nodal

### Mesh:

\[ i_1 + 5i_2 = 10 \quad (1) \]
\[ -i_1 + i_2 = 2V_\alpha \quad (2) \]

\[ -3i_1 + i_2 = 0 \quad (4) \]

From (1) \& (4):

\[ 3i_1 + 15i_2 = 30 \]
\[ -3i_1 + i_2 = 0 \]

\[ 16i_2 = 30 \]
\[ i_2 = 2 \text{ A.} \]

\[ V_\alpha = \frac{5}{8} \text{ V} \]

### Nodal:

\[ \frac{V_1 - 10}{1} - 2V_\alpha + \frac{V_1}{5} = 0 \]
\[ 5V_1 - 50 - 10V_\alpha + V_1 = 0 \]
\[ 6V_1 - 10V_\alpha = 50 \]
\[ 3V_1 - 5V_\alpha = 25 \quad (1) \]
\[ V_\alpha = 10 - V_1 \quad (2) \]

\[ 3(10 - V_\alpha) - 5V_\alpha = 25 \]
\[ 8V_\alpha = 5 \]
\[ V_\alpha = \frac{5}{8} \text{ V} \]
**Mesh**

\[-(0 + \frac{(i_1 - i_3)}{1} + \frac{(i_1 - i_2)}{2}) = 0 \quad \ldots (1)\]

\[\therefore i_2 = 2 \quad \ldots (2)\]

\[\frac{i_3}{4} + \frac{(i_3 - i_2)}{2} + \frac{(i_3 - i_1)}{1} = 0 \quad \ldots (3)\]

\[P_{35} = \frac{|i_2 - i_3|^2}{3} = \quad \text{W} \]

**Nodal**

\[v_1 = 10 \quad \ldots (1)\]

\[1 (v_2 - v_1) + 2v_2 + 3 (v_2 - v_3) = 0 \quad \ldots (2)\]

\[+2 + 3 (v_3 - v_2) + 4 (v_3 - v_1) = 0 \quad \ldots (3)\]

\[P_{3S} = [v_2 - v_3]^2 \cdot 3 = \quad \text{W} \]
14) Write mesh & nodal equation governing the circuit.

Mesh = 3 + 2 = 5
Nodal = 2 + 2 = 4

15) Mesh = 4 + 2 = 6
Nodal = 3 + 2 = 5

**Mesh:**

\[
10 - i_1 - 4i_3 - 5i_x = 0 \quad (1)
\]

\[
i_1 + i_2 = 3 \quad (2)
\]

\[
i_2 + i_3 = \frac{V_{2x}}{2} \quad (3)
\]

**Nodal:**

\[
\frac{V_1 - 10}{1} + \frac{V_1 - 5i_x}{i_1} - 3 - \frac{V_{2x}}{3} = 0 \quad (1)
\]

\[
\frac{V_2}{2} + 3 + \frac{V_{2x}}{3} = 0 \quad (2)
\]

\[
V_{2x} = V_2 \quad (3)
\]

\[
i_x = 10 - V_1 \quad (4)
\]
15) **MESH:**

\[100 - 20i_1 + 10i_2 + 4ui_0 = 5i_4 = 0 \quad (1)\]

\[-10i_1 + 20i_2 + 4ui_0 = 0 \quad (2)\]

\[i_4 - i_3 = 2 \quad (3)\]

\[i_1 - i_2 = i_0 \quad (5)\]

\[i_1 - i_3 = 2V_0 \quad (4)\]

\[V_0 = 10i_2 \quad (6)\]

**NODAL:**

\[\frac{V_1 - 100}{10} + \frac{V_1 - V_2}{10} + \frac{V_1 - V_3}{10} = 0 \quad (1)\]

\[\frac{V_2 - V_1}{10} + \frac{V_2}{5} + 2V_0 - 2 + \frac{V_3 - V_1}{10} = 0 \quad (2)\]

\[V_2 - V_3 = 4ui_0 \quad (3)\]

\[V_0 = V_1 - V_2 \quad (7)\]

\[i_0 = \frac{V_1 - V_3}{10} \quad (5)\]

---

16) **MESH:**

\[i_1 + 10 - 2i_2 = 0 \quad (1)\]

\[+i_1 + i_2 = -2 \quad (2)\]

\[8i_3 + 5i_4 = 10 \quad (3)\]

\[5i_3 + 9i_4 + 5 = 0 \quad (4)\]

**NODAL:**

\[\frac{V_1 - V_2}{4} + 2 + \frac{V_1}{2} = 0 \quad (1)\]

\[V_2 = 10 \quad (2)\]

\[\frac{V_3 - V_2}{3} + \frac{V_3}{5} + \frac{V_3 + 5}{41} = 0 \quad (3)\]
**Mesh:**
\[\begin{align*}
i_1 - 2i_2 &= -10 \\
i_1 + i_2 &= -2 \\
-3i_2 &= -8
\end{align*}\]
\[\begin{align*}
i_2 &= \frac{8}{3} \text{ A} \\
i_1 &= \frac{21}{3} \text{ A} \\
40i_3 + 25i_4 &= 50 \\
40i_3 + 72i_4 &= -40 \\
-48i_4 &= 90
\end{align*}\]
\[\begin{align*}
i_4 &= -\frac{90}{48} \text{ A} \\
i_3 &= \frac{15}{48} \text{ A}
\end{align*}\]

**Nodal:**
\[v_1 - 10 + 2 + \frac{v_1}{2} = 0\]
Theorem 1:

**Source Transformation Technique:**

1. \[ i = \frac{1 \times 8}{18 + 12 + 6} = \frac{1}{4} \]  
   \[ i = \frac{8}{5} \text{ A} \]

2. **Diagram and Analysis**
3) Find $V_o$ in one step.

$$V_o = \Theta 50 \left( \frac{4}{4i + 6} \right) = -20 \text{ V}$$

4) Find $i_x$ using S.T.I.

$$i_x = \frac{10 + 2i_x}{3} \Rightarrow i_x = 10 \text{ A}$$

5) If diode is ideal one, find current through it.

$$0 \Rightarrow I_D = \frac{2}{2} = 1 \text{ A}$$
Theorem 2:---

Superposition Theorem:---

In any linear, active, bilateral n/w, consisting of no. of energy sources, resistances, etc.; the effect produced in any element when all sources act at a time is equal to sum of effect in same element when each source is considered independently.

1) The no. of sub-circuits to be solved by applying S.P.T. is __________

⇒ Sum of Independent sources only.

2) Which of the following electrical parameter cannot be directly evaluated by using S.P.T?

(a) Voltage (b) Current
(c) Power (d) Charge.

⇒ non-linear electrical parameter

⇒ While applying S.P.T., we consider only 1 independent source in every sub-circuit when other nly sources are replaced by short circuit & ideal current sources are replaced by open circuit.

However dependent sources cannot be suppressed.
3) \[ \text{Find } i \text{ using S.P.T.} \]

\[ i_1 = \frac{10}{3} \, \text{A} \]

\[ i_2 = 2 \left( \frac{1}{3} \right) = \frac{2}{3} \, \text{A} \]

\[ i = i_1 + i_2 = \frac{10}{3} + \frac{2}{3} = \frac{12}{3} = 4 \, \text{A} \]

4) \[ \text{Find } V_x \text{ with help of S.P.T.} \]

\[ \text{Step 1 (10 V only)} \]

\[ V_x = 10 \left( \frac{2}{2 + \sqrt{3}} \right) = 6 \, \text{V} \]

\[ \text{Step 2 (5 A only)} \]

\[ V_x = -2 \left[ \delta \left( \frac{3}{3 + 2} \right) \times \frac{1}{2} \right] = -3 \, \text{V} \]

\[ V_x = V_x + V_x'' = 6 - 3 = 3 \, \text{V} \]

\[ \text{Check (Nodal): } \]

\[ V_1 = 10 \quad \rightarrow \quad 0 \]

\[ \frac{V_2 - 10}{2} + \frac{V_2}{2} + \frac{V_2 - V_3}{1} = 0 \quad \rightarrow \quad 2V_2 - 2V_3 = 10 \]

\[ -3V_2 + 4V_3 = -5 \quad \rightarrow \quad 3 \]

\[ 5V_2 = 15 \]

\[ V_2 = 3 \quad \text{so} \quad V_2 = 3 \]
5) - Find \( i_x \) using S.P.T.

**Step 1** (10 V only)
\[
i_x' = 10 \times \frac{1}{1+1} = 5 \text{ A}.
\]

**Step 2** (5 V only)
\[
i_x'' = -5 \left( \frac{\frac{2}{3}}{\frac{2}{3}+2} \right) = -5 \left( \frac{2}{8} \right) = -1.25 \text{ A}
\]

**Step 3** (3 A only)
\[
i_x''' = 0 \text{ A}
\]

**Check (Not By S.P.T)**
\[
i_x = 5 + 0 - \frac{5}{\frac{15}{4}} = \frac{15}{4} \text{ A}
\]

**Check (Nodal)**
\[
\frac{V_2-10}{1} + \frac{V_2}{2} + \frac{V_2-5}{2} = 0 \quad \Rightarrow \quad 4V_2 = 25
\]
\[
V_2 = \frac{25}{4}
\]

Now:
\[
i_x = \frac{V_1-V_2}{1} = 10 - \frac{25}{4} = \frac{15}{4} \text{ A}
\]

6) - What is the power lost in 5 \( \Omega \) resistor using S.P.T.

**Step 1** - (16 V only)
\[
i_1 = \frac{16}{8} = 2 \text{ A}
\]

**Step 2** - (8 A only)
\[
i_2'' = 8 \times \frac{2}{8} = 2 \text{ A}
\]

**Power is non-linear parameter & hence cannot be calculated directly using S.P.T.**
Step 2: (16A only)

\[ i''' = -16 \times \frac{3}{8} = -6 \text{ A} \]

Using S.P.T

\[ i = i' + i'' + i''' = 2 + 2 - 6 = -2 \text{ A} \]

\[ \Rightarrow \frac{5}{3} \uparrow 2 \text{ A} \]

So,

\[ P_{\text{loss}} = I_{\text{net}} \times R \]

\[ = 4 \times 5 = 20 \text{ W} \]

Check

\[ -16 + (16 + I) + 2(8 + I) + 5I = 0 \]

\[ \Rightarrow 8I = -16 \]

\[ \Rightarrow I = -2 \text{ A} \]

\[ P_{\text{loss}} = 4 \times 5 = 20 \text{ W} \]

Step 1 (10V only)

Find \( i_y \) using S.P.T.

\[ 10 - i_y' - 10i_y' - 2i_y'' = 0 \]

\[ \Rightarrow 13i_y' = 10 \]

\[ \Rightarrow i_y' = \frac{10}{13} \]

Step 2: (2A only)

\[ i_y'' + 10i_y' + 2(i_y' + 2) = 0 \]

\[ \Rightarrow 13i_y'' = -4 \]

\[ \Rightarrow i_y'' = -\frac{4}{13} \text{ A} \]

Step 3: (5V only)

\[ 3i_y''' + 10i_y'' + 5 = 0 \]

\[ \Rightarrow i_y''' = -\frac{5}{13} \text{ A} \]
\[ B_4 \text{ S.P.T.} \]

\[ i_y = \frac{10}{13} - \frac{L_1}{13} - \frac{5}{13} = \frac{-1}{13} \text{ A} \]

**Check**

\[-10 + i_y + 10i_y + 2(z + iy) + 5 = 0 \]
\[13i_y = 1 \]
\[i_y = \frac{1}{13} \text{ A} \]

---

**Find V_{xc} using S.P.T.**

**Step 1 (10 V only)**

\[ V_{x_c}^1 = 10 - 4V_{x_c} \]
\[ V_{x_c}^1 = 2 \text{ V} \]

**Check**

\[ V_{x_c} + 4V_{x_c} - 10 - 20 = 0 \]
\[ 5V_{x_c} = 30 \]
\[ V_{x_c} = 6 \text{ V} \]

**Step 2: (2 A only)**

\[ 4V_{x_c} - 10(z) + V_{x_c}'' = 0 \]
\[ V_{x_c}'' = 4 \text{ V} \]

**By S.P.T.**

\[ V_{x_c} = 4 + 2 = 6 \text{ V} \]

---

**The power lost in resistor**

- **When voltage source alone act**, it is 9 W.
- **When current source alone act**, it is 4 W. What is total power lost in resistor \( R \) when both sources act simultaneously?

(a) 14 W  
(b) 5 W  
(c) 13 W  
(d) 25 W

**V_{alone act}**

\[ P_i = I_i^2 R = 9 \Rightarrow |I_i| = \frac{3}{\sqrt{R}} \]
\[ P_2 = I_2^2 R = 4 \Rightarrow I_2 = \frac{2}{\sqrt{R}} \]

Now,
\[ I_{\text{net}} = I_1 \pm I_2 \]
\[ P_T = \left[ I_{\text{net}} \right]^2 R = \left[ \pm I_1 \pm I_2 \right]^2 R \]
\[ = \left[ \pm \frac{3}{\sqrt{R}} \pm \frac{2}{\sqrt{R}} \right]^2 R = \left[ \pm 3 \pm 2 \right]^2 \text{ W} \]

\[ P_T = 1 \text{ W} \quad (I_1 \text{ & } I_2 \text{ different}) \]

or
\[ 25 \text{ W} \quad (I_1 \text{ & } I_2 \text{ same direction}) \]

\[ V_1 \quad \text{and} \quad V_2 \]

\[ \begin{array}{c|c|c|}
V_1 & V_2 & I \\
\hline
10 & 0 & 5 \text{A} \\
0 & -5 & 1 \text{A} \\
\end{array} \]

If \( V_1 \) alone acts \:
10 \rightarrow 5 \text{A}
15 \rightarrow \frac{15 \times 5}{10} = 7.5 \text{A} \quad \text{(Homogeneity)}

If \( V_2 \) alone acts
-5 \rightarrow 1 \text{A}
15 \rightarrow \frac{15}{-5} = -3 \text{A} \quad \text{(Homogeneity)}

By \( \text{SPR} \)
\[ I = 7.5 - 3 = 4.5 \text{A} \]
If \( I_{S_1} = 10 \text{ A}, \) \( I_{S_2} = 5 \text{ A}, \) then \( V_{oc} = 20 \text{ V} \)

If \( I_{S_1} = 20 \text{ A}, \) \( I_{S_2} = -5 \text{ A}, \) then \( V_{oc} = 10 \text{ V} \)

New: \( I_{S_1} = I_{S_2} = 15 \text{ A}, \) then \( V_{oc} = \) 

\[ V_{oc} = f \left( I_{S_1}, I_{S_2} \right) \]

\[ V_{oc} = \alpha I_{S_1} + \beta I_{S_2} \]

\[
\begin{align*}
20 &= \alpha (10) + \beta (5) \\
10 &= \alpha (20) + \beta (-5) \\
30 &= \alpha (30)
\end{align*}
\]

\[ \alpha = 1 \Rightarrow \beta = 2 \]

If \( V_5 = 30, \) then

\[ I_1 = 10 \text{ A}, \] \[ V_2 = 20 \text{ V}, \] \[ P_3 = 30 \text{ W} \]

If \( V_5 = 50 \text{ V}, \) then

\[ I_1 = ?, \] \[ V_2 = ?, \] \[ P_3 = ? \]

(proportionality, homogeneity, Ohm's law).

\[ I_1 \quad \rightarrow \quad 30 \rightarrow 10 \text{ A} \]

\[ 50 \rightarrow I_{\text{new}} = \frac{50 \times 10}{30} = 16.66 \text{ A} \]

\[ V_2 \quad \rightarrow \quad 30 \rightarrow 20 \text{ V} \]

\[ 50 \rightarrow V_{2\text{ new}} = \frac{50 \times 20}{30} = 33.33 \text{ V} \]

\[ P \propto V^2 \]

\[ (30^2) \rightarrow 30 \]

\[ (50^2) \rightarrow P_{2\text{ new}} = \frac{(50)^2 (30)}{(30)^2} = \frac{250}{3} \text{ W} \]
Theorem 3:—

Thevenin's Theorem:—

In any linear bilateral (active) network consisting of energy sources, resistors, etc., with open o/p terminal defined can be converted into a simple network consisting of voltage source in series with resistance.
Theorem 4:

**Norton's Theorem:**

In any linear, active, bilateral, non-consisting of no. of energy sources, resistors, etc., with open output terminal defined, can be converted into a simple circuit consisting of current source in parallel with resistance.

\[ \begin{align*}
\text{Norton} & \rightarrow \text{Thevenin} \\
\begin{align*}
\text{Norton: } & R_N = R_{th} \\
\text{Thevenin: } & I_N = I_{sc}
\end{align*}
\end{align*} \]

**Thevenin & Norton equivalent are duals of each other.**

i.e. They are source transformable.

**Category 1:** Problems with only independent sources

1) Determine current \( i \) using:
   1. Thevenin th.

\[ R_{th} = 1 \Omega \]

1. **Thevenin th.**
10V
2A
2V
q
V_{TH}
2A

\begin{align*}
\text{KVL:} \\
-10 - 2 + V_{TH} &= 0 \\
&\Rightarrow V_{TH} = 12 \text{ V}
\end{align*}

\[
i = \frac{12}{3} = 4 \text{ A}
\]

\[R_N = R_{TH} = 6 \Omega\]

\[I_N = 10 + 2 = 12 \text{ A}\]

\[
i = 12 \left( \frac{1}{1+2} \right) = 4 \text{ A}
\]

Find \(V_x\)

\[R_{TH} = \left( \frac{11 \Omega}{3} \right) = \frac{4}{2} \Omega
\]

\[\text{Thevenin T.N.}\]

\[\text{Thevenin Resistance} = \frac{4}{2} \Omega
\]
1. **KVL**
   
   \[-10 + 5 + V_{1n} = 0\]
   
   \[\Rightarrow V_{1n} = 5\, V\]
   
   \[V_{3c} = 5 \left( \frac{2}{2 + \frac{4}{3}} \right) = 3\, V\]

2. **Norton's Theorem**
   
   \[R_N = R_{1n} = \frac{4}{3}\, \Omega\]
   
3. **KCL**
   
   \[\frac{5 + V}{4} + \frac{V - 10}{3} = 0\]
   
   \[4\, V = -5\]
   
   \[\Rightarrow V = -\frac{5}{4}\, V\]
   
   \[5 = I_N + \frac{5}{4}\]

   \[I_N = \frac{15}{4}\, A\]

   \[V_{3c} = 2 \left( \frac{15}{4} \left( \frac{4/3}{2 + 4/3} \right) \right) = 3\, V\]

3. **Find \(i_x\) using Thévenin & Norton's Theorem**
1. Thévenin's th.

\[ R_{th} = 2 \Omega \]

\[ R_{th} = 2 \Omega = 1 \Omega \]

\[ KVL \]

\[ 10 - V_{th} - \frac{5}{2} V = 0 \]

\[ \therefore V_{th} = \frac{15}{2} V \]

\[ I_x = \frac{15}{2} A \]

2. Norton's the.

\[ R_N = R_{th} = 1 \Omega \]

\[ KCL \]

\[ I_N = 5 + \frac{5}{2} \]

\[ = \frac{15}{2} A \]

\[ I_x = \frac{15}{2} \left( \frac{1}{2} \right) \]

\[ I_x = \frac{15}{4} A \]

Find power lost in 5 \( \Omega \) resistor, using Thévenin & Norton.
1. **Thevenin**

\[ R_{TH} = 2 + 1 = 3 \Omega \]

\[ V_{TH} = 16 - 16(1) - 2(8) = 16 - 32 = -16V \]

2. **Norton**

\[ S_1: R_N = R_{TH} = 3 \]

\[ S_2 \]

\[ 16 - (I_{n+16}) - 2(I_{n+8}) = 0 \]

\[ 3I_n = 16 - 32 \]

\[ I_n = \frac{-16}{3} A \]

\[ I_{ab} = \frac{-16}{3} \left( \frac{3}{8} \right) = -2 A \]

\[ P_{lost} = (-2)^2 \times 5 = 20 W \]
5) Determining Thevenin & Norton equivalent below

1. Thevenin

\[ R_{TH} = \frac{2}{1} + \left( \frac{6}{13} \right) = \frac{2 + 6}{13} = \frac{8}{13} \Omega \]

\[ V_{TH} = 6 + \frac{18}{9} \left( \frac{9}{2} \right) = 6 + 9 = 18 \text{ V} \]

2. Norton

\[ R_{N} = R_{TH} = 4.5 \Omega \]

Using KCL:

\[ \frac{V - 18}{3} + \frac{V}{6} + \frac{V + 6}{2} = 0 \]

\[ 2V - 36 + V + 3V + 18 = 0 \]

\[ V = 3 \]

Determining Norton & Thevenin equivalent below

A.B.
(1) **Thevenin's**

\[ V_{TH} = 1.2 + 0 + 6 = 8 \text{ V} \]

\[ R_{TH} = \frac{1}{2 + 3 + (3.116)} = 2 + 5 + 2 = 7 \Omega \]

\[ \begin{align*}
& \text{ Norton's:} \\
& R_N = R_{TH} = 7 \Omega \\
& \text{ Now,} \\
& 18 - 9I_1 + 3I_2 = 0 \\
& 3I_1 - I_2 = 6 \quad (1) \\
& 38I_1 - 8I_2 = -12 \quad (2) \\
& \text{ From (1) and (2):} \quad 7I_2 = 18 \Rightarrow I_2 = \frac{18}{7} \text{ A} \\
& \text{ But } I_N = I_2 \Rightarrow I_N = \frac{18}{7} \text{ A} \\
\end{align*} \]

(2) Determine **Thevenin** and **Norton** equivalents across ab.
(1) **Thevenin:**

\[ R_{TH} = 20 \Omega \]

\[ 30 - 30 I_1 + 10 I_2 = 50 \]

\[ I_1 - 3 I_2 = -6 \]

\[ I_2 = \frac{16}{8} = 2 \text{ A} \]

\[ V_{TH} = 0 + 10 \times 2 - 10 = 10 \text{ V} \]

(2) **Norton:**

\[ V_1 - V_2 + \frac{V_1 - 50}{10} + \frac{V_1 - V_2 + 10}{10} = 0 \]

\[ 3V_1 - 2V_2 = 40 \]

\[ V_2 - V_1 + \frac{V_2 - 30}{10} + \frac{V_2}{10} + \frac{V_2 - V_1 - 10}{10} = 0 \]

\[ V_1 = 30 \text{ V} \]

\[ V_2 = 25 \text{ V} \]

\[ V_1 - 2V_1 + 4V_2 = 40 \]
Determine the Thévenin equivalent below Y & Z.

\[ R_{\text{TH}} = \frac{10}{11(3+2+1+4)} = \frac{10}{11} = 0.909 \Omega \]

\[ V_{\text{TH}} = \frac{KCL}{V_{\text{TH}}} \]

\[ V_{\text{TH}} = 20 + \frac{V_{\text{TH}}}{10} \cdot 3 - 2 = 20 \]

\[ 2V_{\text{TH}} = 40 \Rightarrow V_{\text{TH}} = 20 \text{V} \]

Now,

\[ R_{N} = R_{\text{TH}} = 0.909 \Omega \]

\[ I_{N} = 2 + \frac{20}{10} = 4 \text{A} \]

\[ V_{\text{TH}} = 20 \text{V} \]
Category 2: Problems with both independent & dependent sources.

Dependent sources cannot be suppressed directly in terms of their resistances. Hence, finding $R_{TH}$ or $R_N$ is not possible directly.

Hence we use Ohm's law where,

$$R_{TH} = R_N = \frac{V_{oc}}{I_{sc}}$$

at target terminal.

Find current $I$ using Thevenin & Norton's th.

$$\frac{V_{oc} - 10}{2Vx} = 0$$

$$V_{ox} = 10 - V_{oc} \quad \text{(i)}$$

$$V_{oc} - 10 - 20 + 2V_{oc} = 0$$

$$V_{oc} = 10 \text{ V}$$

$$V_{x} = \frac{10}{7}$$

But $I_{sc} = 3V_{x}$

$$I_{sc} = 3 \times \frac{10}{7} \Rightarrow I_{sc} = \frac{30}{7} \text{ A}$$
\[ R_{TH} = R_N = \frac{V_{oc}}{I_{sc}} = \frac{10 \times 7}{30} = \frac{7}{3} \Omega \]

\[ i = \frac{10}{3 + \frac{7}{3}} = \frac{15}{8} \text{ A} \]

\[ i = \frac{30}{7} \left[ \frac{\frac{7}{3}}{3 + \frac{7}{3}} \right] = \frac{15}{8} \text{ A} \]

2. Find Thevenin & Norton equiv. below a & b

\[
\begin{align*}
\text{Norton} & \quad V_{oc} \cdot 6 + 5V_{x} + \frac{V_{oc}}{2} = 0 \\
& \quad 2V_{oc} - 12 + 10V_{x} + V_{oc} = 0 \\
& \quad 3V_{oc} + 10V_{x} = 12 - (i) \\
\text{KVL} & \quad -V_{x} + 5V_{x} + V_{oc} = 0 \\
& \quad V_{oc} = -4V_{x} \quad \text{(2)} \\
\end{align*}
\]

From (0 & 2): \(-12V_{x} + 10V_{x} = 12\) \quad \Rightarrow \quad \boxed{V_{oc} = 2V_{x}}

\[
\begin{align*}
\text{KVL} & \quad 6 - 2I_{s} - 5V_{x} = 0 \\
& \quad 2I_{s} = 6 - 5V_{x} \quad \text{(1)} \\
\text{KVL} & \quad -V_{x} + 5V_{x} = 0 \quad \Rightarrow \boxed{V_{x} = 0}
\end{align*}
\]
From \( V_0 \) & \( V_1 \): \( 2I_{sc} = 6 \Rightarrow I_{sc} = 3 \) A

\[ R_{TH} = R_W = \frac{V_{oc}}{I_{sc}} = \frac{24}{3} = 8 \Omega \]

3)

Determine Thévenin & Norton equivalents across the load.

\[ 10i + = 60i + 50 \]
\[ \therefore i = \frac{31}{2} \text{ A} \]

\[ V_{oc} = 50 + 40(-1) = 10 \text{ V} \]

\[ V_{CL} \]
\[ \frac{S}{S} = I_{sc} + \frac{S}{S} \]
\[ \therefore I_{sc} = \frac{5}{8} \text{ A} \]

\[ R_{TH} = R_W = \frac{V_{oc}}{I_{sc}} = \frac{60 	imes 8}{5} = 16 \Omega \]

\[ 10 \]
Find current through 1 Ω resis. using Norton theorem & verify directly

\[ i_0 = 2 \text{ A} \]
\[ V_{oc} = 4 (2) - 10 (2) = -16 \text{ V} \]
\[-10i_0 - 2(2-i_0) + 4i_0 = 0\]
\[-4i_0 = 4\]
\[-i_0 = -1 \text{ A}\]
\[ I_{sc} = 2 - i_0 = 2 + 1 = 3 \text{ A}\]

\[ R_N = \frac{+V_{oc}}{I_{sc}} = -4 \Omega \]
\[ \Rightarrow R_N = -4 \Omega \]

3 \text{ \uparrow}

\[ x = \frac{3 \times \frac{4}{1}}{1} \pm \frac{17}{5} \text{ A}\]

\[ i = 3 \times \left( \frac{-4}{-4+1} \right) = 4 \text{ A}\]

Direct approach:

\[ -4i_0 + 10i_0 + 3(2-i_0) - 0\]
\[-3i_0 = -6\]
\[-i_0 = -2\]
\[ I_{12} = 2 - (-2) = 4 \text{ A}\]

In above problem, \( R_{th} \) or \( R_N \) is -ve

-ve resistance is a very powerful way of modelling active element, or active n/w's in circuit analysis.

eg: transistor as an amplifier is active,
thyristor is considered as very high current gain device, optocouplers, etc.

The V-I charac. of this will be in quadrant 2.
Hence it is active.

Category 3: Problems with only dependent sources:

Such will cannot function on their own as there is no independent active element to drive it.

In thevenin's equivalent $V_{TH} = 0$

In Norton's equivalent $I_N = 0$

But they have only resistance.

This resis. can be obte by indirecly determined by ohm's law by externally exciting them where

$$R_{TH} = R_N = \frac{1}{\frac{1}{V}} = \frac{V_T}{I_A}$$
But such models physically represent our new & electric devices.

1. 1-Parameter equivalent of BJT a common emitter amp?

2. Piece-wise PSPICE & MATLAB models of electronics devices, etc.

1. Determine Thevenin & Norton equiv.

\[ b/w \ a \ & \ b. \]

\[ \begin{align*}
-1 + (1 + i_T + 3(2V_0 + i_T)) &= 0 \\
7i_T + 6V_0 &= 1 \\
V_0 &= 2(V_0 + i_T) \\
3V_0 + 2i_T &= 0
\end{align*} \]

\[ i_T = \frac{1}{3} \]

\[ R_{TH} = R_N = \frac{1}{\frac{1}{3}} = 3 \ \Omega \]

2. \[ 10i_x + 4 - 4i_x - 2i_x = 0 \]

\[ i_x = -1 \ A \]

\[ V_T = 1 + 2(-1) = -1 \ V \]

\[ R_N = \frac{-1}{1} = -1 \ \Omega \]
3) \[ \frac{1}{102} + \frac{1}{80} = \frac{1}{400} + i_T \]

\[ i_T = \frac{1}{80} + \frac{1}{102} - \frac{1}{400} \]

\[ = 0.0198 \]

\[ R_{tn} = R_N = \frac{1}{i_T} \approx 50 \Omega \]

**Special Models:**

4) \[ \frac{i_n}{2} \]

\[ 10V \]

\[ 2 \]

\[ 3A \]

\[ \text{Find } i_X \text{ using Thevenin and Norton's theorem.} \]

\[ -10 + V_{oc} + 3 = 0 \]

\[ V_{oc} = 7, \ V \]

\[ -10 + (I_{sc} + 3) + 2I_{sc} = 0 \]

\[ 3I_{sc} = 7 \Rightarrow I_{sc} = \frac{7}{3} \ A \]

\[ R_{tn} = R_N = \frac{1}{I_{sc}} \times 3 = 3 \Omega \]

\[ i_X = \frac{7}{5} \ A \]

\[ 7V \]

\[ \frac{1}{3} \]

\[ 2 \]

\[ \text{or} \]

\[ i_X = \frac{7}{3} \left( \frac{3}{5} \right) = \frac{7}{5} \ A \]
Determine thevenin equivalent b/w a & b

\[ V_{oc} = 2k \left( \frac{V_{oc}}{4000} \right) + 4 \]

\[ 2V_{oc} = V_{oc} + 8 \quad \Rightarrow \quad V_{oc} = 8 \text{ V} \]

\[ \frac{V_{oc}}{I_{sc}} = 4 - 5I_{sc} = 0 \]

\[ I_{sc} = \frac{4}{5} \text{ mA} \]

\[ R_{\text{th}} = \frac{V_{oc}}{I_{sc}} = \frac{8 \times 5}{4} = 10 \text{ kΩ} \]

---

**Theorem 5:**

**Maximum Power Transfer Theorem:**

In any linear active bilateral network consisting of no. of energy sources with their internal resistances, maximum power is transferred to the load when the load resistance is equal to its equivalent resistance as seen by the load into the supply circuit.

It is an indirect application of Thevenin theorem in designing the electrical loads...
to extract maximum power from source.

\[ I_L = \frac{V_S}{R_S + R_L} \]
\[ P_L = I_L^2 \times R_L \]
\[ P_L = \frac{V_S}{(R_S + R_L)^2} \times R_L \]
\[ \frac{dP_L}{dR_L} = 0 \Rightarrow V_S^2 \left[ \frac{(R_S + R_L)^2 \cdot 1 - R_L^2 \cdot (R_S + R_L)}{(R_S + R_L)^4} \right] = 0 \]
\[ \Rightarrow (R_S + R_L)^2 = R_L \cdot 2 \cdot (R_S + R_L) \]
\[ \therefore R_L = R_S \]

\[ \rightarrow P_{\text{max}} = \frac{V_S^2}{4R_S} \text{ W} \]

In general:

\[ R_L = R_{\text{th}} \& \]

Reduce it to T.E.

\[ P_{\text{max}} = \frac{V_{\text{th}}^2}{4R_{\text{th}}} \text{ W} \]

\[ \text{During } P_{\text{max}} \text{ transfer do the load, the o/p efficiency is 50%} \]
What is the value for which maximum power is transferred to load $R_L$?

$$R_{TH} = (112) + (214)$$
$$= \frac{3}{112} + \frac{25}{12}$$
$$= \frac{21}{10} \Omega$$

What is the maximum power transferred to the load.

$$R_{TH} = \frac{21}{10} = 2.1 \Omega$$

KVL:

$$-V_{TH} = 1\left[20\left(\frac{6}{10}\right)\right] + 2\left[20\left(\frac{4}{10}\right)\right] = 0$$

$$V_{TH} = -12 + 16$$

$$V_{TH} = 4 \text{ V}$$

$$P_{max} = \frac{V_{TH}^2}{\frac{20}{10} \cdot \frac{20}{10}} = \frac{16 \times 10}{2.1^2} = \frac{40}{2.1} \text{ W}$$
What is the value of \( R_x \) for which \( P_{\text{max}} \) occurs?

\[
R_{\text{th}} = \frac{4(14)}{2 + \frac{6R_x}{6 + R_x}}
\]

\( R_x = 2 \Omega \) for \( P_{\text{max}} \)

Find the maximum power transferred across \( R_L \).

Since the bridge is balanced so current can not flow through \( R_L \). Here \( V_{\text{th}} \) across \( R_L \) will be zero.

What is the maximum power transferred to the \( R \).

\[
R_{\text{th}} = \frac{81}{6} = 2.5 \Omega
\]

\[
V_{\text{th}} \quad \text{(Mesh)}\]

\[
24 - 6I_1 + 6 - 3I_2 = 0
\]

\[
2I_1 + I_2 = 16 \quad \text{--1}
\]
\[2I_1 + I_2 = 6 \quad \text{(2)}\]
\[-I_1 + I_2 = 2 \quad \text{(2)}\]
\[\therefore \quad 3I_1 = 4 \quad \Rightarrow \quad I_1 = \frac{4}{3} \, \text{A}\]

Now \underline{KVL}:
\[-V_{TH} + 6 + 3I_1 = 0\]
\[\therefore \quad V_{TH} = 6 + 3 \left(\frac{4}{3}\right) = 6 + 4 = 10 \, \text{V}\]

\[P_{max} = \frac{V_{TH}^2}{4R_{TH}} = \frac{10 \times 10}{4 \times 2} = \frac{25}{2} \, \text{W}\]

**For what value of \( R \), does the power, \( P_{max} \), get transferred to \( 3 \, \Omega \) resistors?**

(a) 6 \quad (b) 3 \quad (c) 2 \quad (d) infinity

\[\text{Acc. to concept}\]
\[R_{TH} \text{ across } 3\Omega = 3\Omega\]
\[\frac{6}{11} R = 3\]
\[\Rightarrow \quad [R = 3.5\Omega] \times\]

\[\text{The value of } R_L \text{ for which } P_{max} \text{ occurs is } R_L = R_s = 7\Omega\]

\[\text{The value of } R_s \text{ for which } P_{max} \text{ occurs is } R_L = 0\Omega\]
What is the max. power transferred to 10Ω. \( 12 \leq R \leq 100Ω \)

Here \( R = 1 \Omega \)

\[ I_{\text{max}} = \frac{100}{10+1} = \frac{100}{11} \text{ A} \]

\[ P_{\text{max}} = (\frac{100}{11})^2 \times 10 = 826.4 \text{ W} \]

What is the value of \( R \) for which max. power is transferred from source to load.

\[ R = 4.1Ω \]

What is the max. power transferred to \( R \)

\[ V_{OC} = 10V \]

\[ R_{\text{th}} = \frac{V_{OC}}{I_{SC}} = \frac{7}{3} \Omega \]

\[ P_{\text{max}} = \frac{10 \times 10}{\frac{7}{3}} = \frac{75}{7} \text{ W} \]
What resist. connected b/w a & b will draw max. power from it & also find this max. power.

\[ I = \frac{30}{2k} \, A \Rightarrow I_0 = \frac{30}{2k} \left(\frac{18}{3k} \right) = 5 \, mA \]

\[ R_{TH} = \frac{V_{oc}}{I_{sc}} \]

\[ 50I_0 - 20I = 0 \]

\[ 20I = 50 \times 5 \]

\[ I = \frac{25}{2} \, mA \]

\[ R_{TH} = \frac{-1/8}{-1/40} = 5.52 \]

\[ P_{max} = \frac{V_{th}^2}{\eta \times R_{TH}} = \frac{(\frac{18}{20})^2}{\eta \times 5} = 0.781 \, W \]
During Pmax transfer to load \( R_L = R_s \) voltage across load is 50% of supply (Vth).

\[
\text{KCL:} \quad \frac{20-10}{10} = \frac{10}{20} + \left(\frac{10-V_0}{5}\right)
\]

\[
1 = \frac{1}{2} + \frac{10-V_0}{5}
\]

\[
V_0 = 10 - \frac{5}{2} = 7.5 \text{ V}
\]

**Exact Method**

\[I_L = \frac{20 - \left(\frac{1}{5} V_0\right)}{14} = \frac{100 - 4V_0}{14 \times 5} = \frac{50 - 2V_0}{35}\]

\[P_L = (I_L)^2 R + (\frac{41 - V_0}{5}) I_L\]

12) For what value of \( R \), more power is transferred from cell A to cell B?

Shortcut: \[
\frac{20-10}{10} = \frac{10}{20} + \frac{10-5}{5} \Rightarrow R = 10 \Omega
\]

Voltage is divided equally across source and load.
The static VI characteristics of n/w shown in Fig (a) is plotted in Fig (b). What is the max. power that can be drawn from the n/w.

→ 4th quadrant ⇒ active elements are present in n/w.

1st operating point

\[ I = 0, \quad V = 20V \]

\[ \Rightarrow V_{TH} = 20V \]

\[ \therefore \quad P_{max} = \frac{V_{TH}^2}{4ICTH} = \frac{20 \times 20}{4 \times 5} = 20W \]

2nd operating point

\[ I = -4A, \quad V = 0 \]

\[ \Rightarrow V_{TH} = 4 \times R_{TH} \Rightarrow R_{TH} = 5 \]
Theorem 6:

Reciprocity Theorem:

In any linear passive bilateral, if excited with only a single source, the ratio of response to excitation remains constant even if the positions of source and load are interchanged.

\[
\frac{I}{V} = \text{const.} \quad \text{By applying Homogeneity principle} \quad \frac{I_1}{V_1} = \frac{I_2}{V_2}
\]

Note:

This theorem is valid for n/s excited with single source only.

This theorem cannot be applied for n/s with dependent source, since dependent sources can make n/s active.

While writing the Reciprocity n/s of given n/s, ideal independent voltage sources are connected in series to the target branch and ideal independent current sources are connected in parallel to target branch.

Eg: Communication lines
Electrical power Tx n/s.
1) Verify Reciprocity theorem for a net shown below in finding current $I$.

**Step 1**

$\text{Final } I = \left[ \frac{10}{7+6}\right] \left( \frac{G}{2+6} \right)$

- Total current
  - $\frac{10}{7+\frac{12}{8}} \times \frac{6}{8} = \frac{15}{17} \text{ A}$

2) **Reciprocal n/w**

$I = \left[ \frac{10}{2+7}\right] \left( \frac{G}{G+7} \right)

= \frac{10}{2+\frac{42}{12}} \left( \frac{6}{12} \right) = \frac{15}{17} \text{ A}$

- Verify R.T.
  - $V_0$ & find $V_0$

(7112) + 1 = $\frac{23}{9}$

- $i_1 = 20 \left[ \frac{2}{2+\frac{23}{9}} \right]

- $V_0 = 4 \cdot i_2$

  - $i_2 = \left[ 20 \times \frac{2}{2+\frac{23}{9}} \right] \times \frac{2}{9}

  - $V_0 = 4 \cdot \left[ 20 \times \frac{2}{2+\frac{23}{9}} \right] \times \frac{2}{9}$

- reciprocal n/w

- $3 + \left[ 21\frac{3}{5} \right] = 3 + \frac{6}{5} = \frac{21}{5}$

  - $i_1 = 20 \left( \frac{4}{21\frac{3}{5} + 4} \right)$

  - $i_2 = 20 \left[ \frac{4}{4 + 21\frac{3}{5}} \right] \left[ \frac{2}{4+3} \right]$
\[ V_0 = i_2 R = 20 \left[ \frac{4}{21 + 21/5} \right] \left[ \frac{2}{5} \right] \times 2 \]

\[ = \frac{20 \times 2 \times 4 \Omega}{\frac{41}{5} \times 5} = \frac{320}{41} V \]

3) Use the data given in fig. A to find current \( I \) in fig. B.

\[ \text{fig (A):} \]
\[ \text{fig (B):} \]

\[ \text{Reciprocal}\]
\[ \text{Applying homogeneity principle}\]

\[ \Rightarrow I = -4 A \]

4) Use the data given in fig (A) to find current \( i \) in fig. (B).

\[ \text{fig (A):} \]
\[ \text{fig (B):} \]

To solve fig. B use superposition theorem.
Step 1

\[ i' = 2/4A = 1A \text{ (from fig A)} \]

(only 5V)

Step 2

\[ 2A \text{ from reciprocity theorem and homogeneity principle applied in fig A.} \]

\[ i'' = 4A \]

\[ i = i' + i'' = 1 + 4 = 5A \]

(only 10V)

5) Use the data in fig A to find current \( i'' \) in fig B.

Apply superposition

\[ i' = 8A \text{ (from fig A)} \]

By homogeneity principle
Step 2:

\[ i'' = -12 \text{A} \]

\[ i = i' + i'' = 8 - 12 = -4 \text{A} \]

Theorem 7:

**Tellegen's Theorem:**

This theorem is verification of law of conservation of energy. However in any linear time invariant system, power at an instant is like an energy over a period. So in this theorem we need to verify:

\[ \sum_{k=1}^{n} V_k I_k = 0 \]

where, \( n \rightarrow \) no. of elements in \( n/w \)

\[ \text{i.e.} \quad \sum V I \text{ | source } = \sum V I \text{ | sinks} \]

1) Verify Tellegen's theorem for the circuit shown below.
Mesh:

1. \( i_1 + 2(i_1-i_3) + 2i_x + 3i_2 = 0 \)
2. \(-i_1 + i_2 = 0\)
3. \(i_3 = -3V_x\)
4. \(i_x = -i_1\)
5. \(V_x = 3i_2\)

Nodal:

1. \(\frac{V_1}{1} + \frac{V_1-V_2}{2} - 3V_x = 0\)
2. \(\frac{V_2-V_1}{2} + \frac{V_3-10}{3} + 3V_x = 0\)
3. \(V_2 - V_3 = 2i_x\)
4. \(i_x = \frac{V_1}{1}\)
5. \(V_x = V_3\)

\[i_1 = -\frac{105}{11} \text{ A}\]
\[i_2 = \frac{5}{11} \text{ A}\]
\[i_3 = -\frac{45}{11} \text{ A}\]
\[V_x = \frac{15}{11} \text{ V}\]
\[i_x = \frac{105}{11} \text{ A}\]

\[V_1 = 105/11 \text{ V}\]
\[V_2 = 225/11 \text{ V}\]
\[V_3 = 15/11 \text{ V}\]

\[\sum P_{\text{source}} = \left(\frac{225}{11} \times 10\right) + \left(\frac{270}{11} \times \frac{45}{11}\right) = 304.95 \text{ W}\]

\[\sum P_{\text{sink}} = \left(\frac{105}{11} \times \frac{105}{11}\right) + \left(\frac{120}{11} \times \frac{60}{11}\right) + \left(\frac{15}{11} \times \frac{50}{11}\right) + \left(\frac{180}{11} \times \frac{45}{11}\right) + \left(\frac{210}{11} \times \frac{50}{11}\right)\]

\[= 304.95 \text{ W}\]
Theorem 8:

**Milliman's Theorem:**

[Parallel Generator Theorem]

\[
V' = \frac{\sum_{i=1}^{n} \frac{V_i}{R_i}}{\sum_{i=1}^{n} \frac{1}{R_i}} = \frac{\sum_{i=1}^{n} V_i G_i}{\sum_{i=1}^{n} G_i}
\]

\[
R' = \frac{1}{\sum_{i=1}^{n} \frac{1}{R_i}} = \frac{1}{\sum_{i=1}^{n} G_i}
\]

**Dual of Milliman's theorem**

\[
I' = \frac{\sum_{i=1}^{n} I_i R_i}{\sum_{i=1}^{n} R_i}
\]

\[
R' = \frac{1}{\sum_{i=1}^{n} R_i}
\]
SUBJECT NAME

ELECTRIC CIRCUITS / ELECTRIC CIRCUITS & FIELDS (E Engineering)
NETWORK THEORY / NETWORK ANALYSIS & TRANSMISSION LINES (E & T Engineering)

IMPORTANCE of SUBJECT

Unique Order of covering IFS Syllabus

1) Fundamentals - Definitions, Notations, Symbols, Units, Formulas, Examples and Applications
2) DC Circuit Analysis - Resistor as Fundamental Component (MESH and NODAL Analysis)
3) DC Network Theorems and Applications
4) Inductors and Capacitors
5) AC Fundamentals - PHASOR, j-Operator, RMS and Average values of Time-Varying Waveforms
6) Concept of POWER in AC, AC Circuit Analysis (MESH and NODAL Analysis)
7) AC Network Theorems and Applications
8) Locus Diagrams, Duals and Duality in Electrical Networks
9) Resonance
10) Magnetic Circuits
11) Network Topology / Graph Theory
12) Transient Circuit Analysis (Time-Domain)
13) Solution of Network Equations using Laplace Transform
14) Network Functions and Filters Concepts
15) Two-Port Networks
16) Network Synthesis (for IES Only)*
17) State Equations for Networks (for IES Only)*
- Three Phase Circuits (for EE Only)**

Reference Books

IES
- Rank-1: Engineering Circuit Analysis, by William Hyat and Kemmerly, TMH Publications
- Rank-3: Network Analysis, by M E Van Valkenburg, PHI Publications
- Rank-4: Networks and Systems, by D Roy Choudhury, New Age International Publications
- Rank-5: Linear Circuit Analysis, by DeCarlo and Lin, OXFORD University Press

"If I HEAR, I will FORGET"
"If I SEE, I will REMEMBER"
"If I DO, I will UNDERSTAND"

IES = FUNDAMENTALS + CONFIDENCE
1. \[ V' = \frac{10 + \frac{20}{2} - \frac{30}{3} + \frac{40}{4}}{1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}} \]
\[ = 9.61 \text{ V} \]
\[ \therefore i_x = \frac{9.61}{5.48} = 1.75 \text{ A} \]

2. \[ R' = \frac{1}{1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}} \]
\[ = 0.485 \Omega \]
Theorem 9:—

Compensation theorem:—

→ This theorem allows us to calculate the correct value of electrical parameters such as voltages & currents when they are subjected to parametric variations within the circuit.

→ This theorem is exclusively used to determine steady state error in measuring instruments as practical meters with their internal resistances will alter the ideal values when they are connected into the circuit.

On placing ammeter to measure current its internal resistors also gets added causing $I'$ to flow.

Removal of sources will cause $V_C = I [AR]$ compensated $n/w$. 
1) Find the change in current introduced by ammeter with an internal resist of 0.15 Ω while measuring the current in 6 Ω resist branch. Also determine steady state error introduced by the meter.

**Step 1:** Find 'I' theoretically

\[
I = \frac{30}{2+2} \times \frac{3}{9} = \frac{5}{2} \text{ A}
\]

**Step 2:** Calculate compensated V(t)

\[
V_c = I \times [AR] = \frac{5}{2} \times (0.15) = 0.375 \text{ V}
\]

**Step 3:** Compensated V(t) =

\[
I_c = \frac{(0.375)}{6.15 + (2.113)} = \frac{0.375}{7.38} = 0.051 \text{ A}
\]

So by connecting an ammeter with internal resist of 0.15 Ω, the current in the 6 Ω branch is reduced by 0.051 A.

Now, % Error = \[
\frac{I - I'}{I} \times 100\% = \frac{I_c}{I} \times 100\%
\]

\[
= 0.051 \times 100\% = \frac{2.05\%}{L} \text{ acceptable}
\]
**Theorem 10:**

**Substitution theorem:**

In any linear active unilateral n/w consisting of no. of energy sources, passive elements, etc., any passive element can be substituted in terms of its equivalent voltage and current for further analysis of n/w w/o disturbing the rest of the n/w provided the power absorbed by this passive element & its equivalently substituted source is same.

\[
\text{P}_{\text{absorbed}} = \frac{V^2}{R} = I^2R = V \cdot I
\]

We can model a semiconductor device as a passive element while conducting in terms of its on-state voltage drop, using substitution theorem.

\[
\text{Ron} \quad (\text{dynamic on-state resist.}) \quad \frac{1}{I_D} \quad \frac{1}{V_T} \quad \text{ON-state voltage drop.}
\]
If voltage drop across 2 \( 5 \Omega \) resistors is 20V, the 5 \( \Omega \) resists branch below can be substituted by an equivalent voltage of _______ 

\[ P_{\text{absorbed}} = (8)^2 \times 5 \]

\[ = 125 \text{ W} \]

2) Use substitution theorem to substitute 1 \( 5 \Omega \) branch in 5 different ways (at least)

3) \[ P_{\text{abs}} = 1 \text{ W} \]

4) \[ P_{\text{abs}} = (2)^2 \times \frac{1}{2} = 2 \text{ W} \]

5) \[ P_{\text{abs}} = (1.5)^2 (0.1) + 0.9 \]

\[ = 1 \text{ W} \]
3) For the balanced bridge determine the value of $I_S$.

$$i = \frac{V}{R} = \frac{5}{1k} = 5\,mA$$

So, $I_S = 2 \times i = 2 \times 5\,mA = 10\,mA$.

4) Use substitution two, to find the value of voltage $V_a$ for which max. power is transferred to the load.

$$R_{\text{eq.(load)}} = \frac{10 \times 5}{10 + 2R}$$

$$25 + R = 10 + 2R$$

$$R = 15\,\Omega$$
5) **What is applicable?**
   a) T.E only  
   b) N.E only  
   c) Both  
   d) None

**Thevenin Theo.**

\[ R_{TH} = R_N = \infty \]

\[ V_{TH} \rightarrow \text{cannot be determined} \]

**Norton Theo.**

\[ I_N = I \]

\[ R_N = \infty \]

\[ (\text{fig a}) \]

Her Thevenin Theo. is equal.

**is possible.**

6) \[ V \]

\[ R_{TH} = R_N = 0.5 \Omega \]

\[ V_{TH} = V \]

\[ I_N \rightarrow \text{cannot be determined} \]

7) \[ \begin{array}{c}
1V \text{ delivers } 0W \\
1A \text{ delivers } 1W \\
1\Omega \text{ absorbs } 1W
\end{array} \]

8) The new N consists of only the resistor. Use the data given in fig (a) to find current I in fig (b)

\[ \begin{array}{c}
R_{TH} = 10/5 = 2 \Omega \\
(\text{fig b})
\end{array} \]
Using reciprocity:

When no resistance is present, we get 6 A current (using homogeneity). When Rin does not appear

\[ I = 6 \left( \frac{2}{3} \right) = 4 \text{ A} \]

Rin appears. With addition of Rin.

Addition of \( R \) resistance will allow it to appear at input/port resistance.
PROPERTIES OF INDUCTORS

1. Sine \( V = L \frac{di}{dt} \)

2. For dc excitation, \( \frac{di}{dt} = 0 \), \( V_L = 0 \) \( \rightarrow \) inductor is s.c. for ideal D.C.

3. An inductor never allows sudden change in current through it.

4. This is the principle of operation of choke coil in fluorescent lamp.

If inductor really allows sudden change in current through it, we get huge impulse voltages appear across it.

5. An ideal inductor is a coiled wire with zero internal resistance. So power dissipated is zero. \( L(\frac{di}{dt})^2 = 0 \)

6. Practical inductors will have small internal resistance & they are represented as coil shaven behind which allows some power losses.

7. Inductors are available in different shapes & sizes & they classified on the basis of the type of core material on which winding is done.
6. Inductors are used as filters, current limiting reactors, var compensators, etc in communication & power systems.

**Properties of Capacitors**

Since \( i = C \frac{dV}{dt} \)

1. For DC excitation, \( \frac{dV}{dt} = 0 \), \( i_c = 0 \)
   \[ \rightarrow \text{Capacitor acts as open circuit for ideal D.C. (in steady state)} \]

2. Capacitor never allows sudden change in voltage across it.

3. Ideal capacitors are considered to have infinite dielectric capacitance between the electrodes, so dielectric losses are zero. Electric conduction is through polarization.

\[ R \rightarrow \infty \]

4. Practical capacitors are considered to have very large dielectric resistances between the electrodes, so they undergo losses.

5. Capacitors are available in different shapes and sizes and they are classified on the basis of dielectric material between the electrodes.

6. They are used as filters, compensators, power factor correcting equipments.
Signal conditioning & wave shaping, etc
in communication & power sys.

\[ L_{eq} = 5 + \left[ \frac{[(3+18)+2]}{4} \right] \]
\[ = 5 + \left[ \frac{23}{4} \right] \]
\[ = 5 + 5.75 \]
\[ = 10.75 \text{ mH} \]

\[ L_{eq} = 6 + \left[ \frac{6 + 12 + 11 + 12}{4} \right] \]
\[ = 6 + 9 \]
\[ = 15 \text{ mH} \]

\[ L_{eq} = \frac{L}{2} + \frac{L}{2} + \frac{L}{2} \]
\[ = \frac{3L}{2} \]

\[ L_{eq} = \frac{5L}{3} + \frac{L}{3} \]
\[ = \frac{5L^2}{3} + \frac{L^2}{3} \]
\[ = \frac{5L^2}{3} + \frac{8L^3}{3} \]
\[ = \frac{5L}{8} \]
Mesh

\[-V + 5 \frac{di}{dt} + 2i \left[ \frac{di}{dt} - \frac{di}{dt} \right] = 0\]

\[q \times -4y = V \quad \text{(1)}\]

\[4 \left[ \frac{di}{dt} - \frac{di}{dt} \right] + 2 \frac{di}{dt} + 3 \frac{di}{dt} = 0\]

\[-2x + 7y = 0 \quad \text{(2)}\]

\[X \left[ q - 4 \times \frac{2}{7} \right] = V\]

\[V = \left[ \frac{63 - 8}{7} \right] X \Rightarrow \frac{55}{7} \frac{di}{dt}\]

\[\Rightarrow \text{Leq} = \frac{55}{7} \text{H}\]

Find the steady state current through the inductor & also energy stored in it.

Scaling \(L\) by \(10 \text{mH}\)
\[ \text{eq} = \left[ \frac{1}{3} + \left( \frac{9}{2} \cdot 11 \cdot 6 \right) \right] \times 10 \text{ mH} \]

\[ = 2.9 \times 10 \text{ mH} \text{ (scaled product)} \]

\[ \Rightarrow i_{1m} = 15 \text{ A} \]

\[ E_L = \frac{1}{2} (1 \text{ mH}) (15)^2 = 112.5 \text{ mJ} \]

\[ \Rightarrow i_{2m} = 15 \text{ A} \]

\[ E_L = \frac{1}{2} (2 \text{ mH}) (15)^2 = 225 \text{ mJ} \]

\[ \Rightarrow i_{3m} = 0 \text{ A} \]

\[ E_L = \frac{1}{2} (3 \text{ mH}) (10)^2 = 150 \text{ mJ} \]

These energies are in electromagnetic form (DC flux)

7) If the current flowing through 2 H inductor is as shown, plot the voltage across it.

\[ i(t) \]

\[ 14 \text{ A} \]

\[ 0 < t < 2 \text{ m} \]

\[ i(t) = 7t \Rightarrow v = 2 \frac{d}{dt} (7t) = +14 \text{ V pulse} \]

\[ 2 \text{ m} < t < 4 \text{ m} \]

\[ i(t) = 14 \Rightarrow v = 2 \frac{d}{dt} (14) = 0 \text{ V} \]

(for DC L is shorted)
\[ 4 \text{m} < t < 11 \text{m} \]

\[ i(t) = -2t + c \Rightarrow v = 2 \frac{d}{dt} (-2t + c) \]

\[ = -4(V) \]

[Graph showing voltage and time (ms)]

\[ \text{Current flowing through 2H inductor } \]

\[ t(\text{ms}) \text{ is shown.} \]

\[ \text{Flat voltages across inductor.} \]

[Graph showing current and time (ms)]

\[ \text{Numerators restrict magnitude}\]

\[ v = \frac{\sum dI}{\Delta t} \]

[Impulse response]

\[ \downarrow \text{Impulse instruction} \]

\[ \Delta V = \int \frac{\sum \Delta I}{\Delta t} \]

[Impulse]

\[ \frac{\Delta I}{\Delta t} \rightarrow 0 \]
\[ V = 2 \left[ \frac{2-0}{\Delta t} \right] = 4V \]  
\[ \text{impulse} \]
\[ \Delta t = 3 \text{ msec} \]
\[ V = 2 \left[ \frac{-2-2}{\Delta t} \right] = -8V \]
\[ \text{impulse} \]
\[ \Delta t = 6 \text{ msec} \]
\[ V = 2 \left[ \frac{2-(-2)}{\Delta t} \right] = +8V \]

4) A practical coil lies on an inductance of 2H & resistance of 1.5Ω. If this coil is excited with the current as shown below, find the total energy absorbed by the coil up to 1st 4 seconds.

\[ E_{\text{abs}} = E_{\text{dissipated}} + E_{\text{stored}} \]

\[ E_{\text{diss}} = \sum_{i} R \Delta t = \int_{0}^{4} [i(t)]^2 R \, dt \]
\[ = \int_{0}^{4} (3t^3 \times 1.5) \, dt + \int_{0}^{4} 2 \, dt \]
\[ = 3 \left[ \frac{t^4}{4} \right]_0^4 + 3[4-2] \]
\[ = 24 + 72 \]
\[ = 96 \text{ J} \]
\[ E_{\text{stored}} = \int P_L \, dt = \int L \frac{di(t+)}{dt} \, dt + \int 2 \cdot 6 \, \frac{dI(0)}{dt} \, dt. \]

\[ = 9 \left[ t^2 \right]_0^6 = 36 \, J \]

\[ E_{\text{absorbed}} = 96 + 36 = 132 \, J \]

**Note**

An inductor stores energy for some time variance occurring at any instance in that instant. It retains this energy as long as excitation is given.

So energy stored here up to the first 4 sec is the energy stored at the 4th sec

\[ E_L = \frac{1}{2} L i^2 = \frac{1}{2} (2) (6)^2 = 36 \, J \]

---

**Find total energy about 6 sec second**

\[ i(t) \]

\[ 0 < t < 2 \quad i(t) = \max = i(t) = 3t \]

\[ 2 < t < 4 \quad i(t) = 6 \]

\[ 4 < t < 6 \quad (i(t) - 0) = \frac{(6-0)}{(4-6)} (t-6) \quad \Rightarrow \quad i(t) = -3t + 18 \]
In the above problem adding the limits from 4 to 6, we get

\[ E_{\text{diss.}} = 96 + \int_{4}^{6} (-3t + 18) \, dt \]

\[ = 96 + \frac{3}{2} \int_{4}^{6} (t^2 - 12 + 36) \, dt \]

\[ = 96 + 24 \]

\[ = 120 \text{ J} \]

\[ E_{\text{stored}} = 36 + \int_{4}^{6} \frac{9}{2} (t^2 - 18) \cdot \frac{dl}{dt} \, dt \]

\[ = 36 + 18 \int_{4}^{6} (-t^2 + 18) (\frac{-1}{2} + \frac{18t}{2}) \, dt \]

\[ = 36 - 36 = 0 \text{ J} \]

\[ E_{\text{absorbed}} = 120 \text{ J} \]

11) \[
\begin{align*}
\text{C}_{\text{eq}} &= \frac{2C \times 2C}{4C} \\
&= \frac{4C^2}{4C} = C
\end{align*}
\]

12) \[
\text{C}_{\text{eq}} = \left[ \frac{(2 + 2) \times 4}{8^2} + 2 \right] \text{ series (} 3 \text{UF) }
\]

\[
= \frac{4 \times 3}{7} = \frac{12}{7} \text{ mF}
\]
13) \[
\begin{align*}
C_{xy} &= \frac{C_s}{C_s + C_{c+}\text{c}} \\
\therefore C_s &= \frac{(C_s + C_c) \cdot \frac{1}{2}}{2}
\end{align*}
\]

14) \[
V_s = (2000) i + \frac{1}{100 \mu \text{F}} \int 50i \text{dt}
\]
\[
\Rightarrow V_s = \frac{2000}{R} i + \frac{1}{2\mu \text{F}} \int \text{dt}
\]
\[
\Rightarrow C_{\text{loop}} = 2 \mu \text{F} \quad \text{and also} \quad R_{\text{loop}} = 2000 \Omega
\]

15) \[
C_x = \frac{1}{C_{cy}} = \frac{1}{1 + \frac{1}{2} + \frac{t \times \frac{1}{3}}{\frac{1}{2}}} = 1 + \frac{1}{3} + \frac{2}{3} = \frac{2}{3}
\]
\[
\therefore C_x = \frac{1}{2} \mu \text{F}
\]

Convert star to delta
\[
\begin{align*}
\frac{1}{C_y} &= \frac{1}{1} + \frac{1}{2} + \frac{t \times \frac{1}{3}}{\sqrt{3}} = 1 + \frac{1}{2} + \frac{3}{2}
\end{align*}
\]
\[
\frac{1}{C_y} = \frac{6}{2} = 3 \Rightarrow C_y = \frac{1}{3} \mu \text{F}
\]
\[
\frac{1}{C_y} = \frac{1}{3} + \frac{1}{2} + \frac{t \times \frac{1}{2}}{\sqrt{1}} = \frac{1}{3} + \frac{1}{2} + \frac{1}{6}
\]
\[
\frac{1}{C_z} = 1 \Rightarrow C_z = 1 \mu \text{F}
\]
\[ C_{eq} = \frac{5 \times \frac{2}{4}}{} = \frac{10}{7} \, \text{F} \]

\[ C_{eq} = 2.23 \times 10 = 22.3 \, \text{mF} \]

If equivalent capacitance bw the plates is \( C \) then \( C_{AB} = \) __________

\[ C_{AB} = 3C \]
1) Three capacitors as arranged in fig. as shown below. The value of volts in bracket indicates their breakdown voltage limit. What is the max volts uptil which each unit n/w can work w/o breakdown of any capacitor. & hence determine max. storage charge in the n/w.

For 10 μF
\[ V = 10 \text{V} \]

For 5 μF
\[ V \left(\frac{2}{5}\right) = 5 \Rightarrow V = 17.5 \text{V} \]

For 2 μF
\[ V \left(\frac{5}{7}\right) = 2 \Rightarrow V = 2.8 \text{V} \]

\[ V_{\text{max}} = 2.8 \text{V} \]

\[ C_{\text{eq}} = \frac{10}{7} + 10 = \frac{80}{7} \text{ μF} \]

\[ q_{\text{max}} = C_{\text{eq}} V_{\text{max}} = \frac{80}{7} \times \frac{2.8}{10} = 32 \text{μC} \]

2) Two capacitors of 1 μF & 2 μF are connected in series across a 30V DC source. Find their steady volts & charge on each. What if these 2 capacitors are disconnected from supply & connected with like polarities together, now determine steady state volts & charge on it each.
\[ V_{1u} = \frac{30(2)}{3} = 20 \text{ V} \]
\[ V_{2u} = \frac{30(1)}{3} = 10 \text{ V} \]

\[ q_{1u} = 4 \times 20 = 80 \text{ mC} \]
\[ q_{2u} = 2 \times 10 = 20 \text{ mC} \]

In current electricity, if current is equal in series connected elements, then in static electricity, the charges will be equal if/when in series connected capacitors.

\[ q_1 = q_2 = 20 \text{ mC} \]

\[ V_1 = V_2 = V \]

\[ \frac{q_1}{C_1} = \frac{q_2}{C_2} \Rightarrow \frac{q_1}{1} = \frac{q_2}{2} \]

\[ q_2 = 2q_1 \]

From law of conservation of charge

\[ q_1 + q_2 = 40 \]

\[ q_1 + 2q_1 = 40 \Rightarrow q_1 = \frac{40}{3} \text{ mC} \]

\[ q_2 = \frac{80}{3} \text{ mC} \]

\[ V_1 = V_2 = \frac{q_1}{C_1} = \frac{q_2}{C_2} = \frac{40}{3} \text{ volts} \]

3) Determine steady state voltage across each capacitor and energy stored in each.
4) A capacitor having a common cross-sectional area $d/2$ m between the plates of a m² & distance $d$ m. The plates are now dipped in ethyl alcohol up to $d/2$ m. What is the ratio of capacitance before and after immersing it into the ethyl alcohol? [Consider $\varepsilon_r = 25$ for ethyl alcohol]

Before: 
$$C = \frac{A \varepsilon_0 \varepsilon_r}{d}$$

After:
$$C' = \frac{A \varepsilon_0 \varepsilon_r}{d/2} \times \frac{A \varepsilon_0 \varepsilon_r}{d/2} = \frac{A \varepsilon_0 [2(\varepsilon_r)]}{d}$$

$$\therefore \frac{C}{C'} = \frac{26}{50}$$
1) Determine steady state voltages across capacitor & current through inductors.

\[ R_T = R_1 + R_2 = 6 \]
\[ i = \frac{30}{6} = 5 \text{ A} \]
\[ i_{L1} = i_{L2} = 5 \text{ A} \]
\[ V_{C1} = -30 + 10 + V_{C1} = 0 \Rightarrow V_{C1} = 20 \text{ V} \]
\[ V_{C2} = 0 \text{ V} \]

2) What is value of \( R \) for which the energy stored in inductor & capacitor are equal.

\[ i_L = 5 \times \frac{2}{R+2} = \frac{10}{2+R} \]
\[ V_C = i_L \times R = \frac{10R}{2+R} \]

Now
\[ E_L = E_C \]
\[ \frac{1}{2} L i_L^2 = \frac{1}{2} C V_C^2 \]
\[ \frac{1}{2} \times \left( 4 \text{ mH} \right) \left( 100 \right)^2 = \frac{1}{2} \left( 160 \text{ mF} \right) \left( 100 \right) \frac{R^2}{(2+R)^2} \]
\[
R^2 = \frac{1}{4} \frac{1}{25} \Rightarrow R = 5 \Omega
\]

**Steady State AC Circuit Analysis:**

- **Radians:**
  \[ V = V_m \sin \omega t \]

- **Degrees:**
  \[ V = V_m \sin 360^\circ \theta \]

\[ V_m \rightarrow \text{amplitude} \]
\[ \omega \rightarrow \text{angular freq.} \quad \text{(rad/sec)} \]
\[ \omega = 2\pi f = \frac{2\pi}{T} \]

\[ I_m \rightarrow \text{Current} \]
\[ V = V_m \sin \left( \frac{2\pi}{T} t \right) \]

- **Power:**
  \[ P = \frac{1}{2} V_m I_m \]
  \[ I = \frac{1}{50} = 20 \mu A \]

\[ 1T = 2\pi = 360^\circ = 20 \mu s. \]
Standard sine wave:

\[ V_x = V_m \sin(\omega t + \phi) \quad ; \quad \phi = \text{phase shift} \ (\text{deg}) \]  
\[ t = \text{time shift} \]

\[ V_1 = V_m \sin(\omega t + \phi) \]

- \( V_1 \) comes early than \( V \) by \( \phi \)°
- \( V_1 \) leads \( V \) by \( \phi \)°
- (+) → leading

\[ V_2 = V_m \sin(\omega t - \phi) \]

- \( V_2 \) lags \( V \) by \( \phi \)°
- (-) → lagging

**India**, \( \phi = 60° \)

360° → 20 msec
60° → \( t_{\text{shift}} \)

\[ t_{\text{shift}} = \frac{\phi}{360} \times 20 \text{ msec} = 3.33 \text{ msec} \]

\[ \omega = 2\pi f = 2\pi (50) = 100\pi \text{ rad/see} \]
represents sinusoids

* Time shift can be easily understood from phase shift (i.e. using phasors). It also indicate the amount of shift.

\[ V = |V_{m}| \angle \theta \]
\[ V_1 = |V_{m}| \angle \phi \]
\[ V_2 = |V_{m}| \angle -\phi \]

S-plane

\[ P = x + iy \rightarrow \text{Rectangular Form} \]
\[ P = r \angle \theta \rightarrow \text{Polar Form} \]
\[ P = r e^{i\theta} \rightarrow \text{Euler's Form} \]
Phasor relationship b/w voltages and currents in passive elements:

Let \( V = V_m \sin \omega t \) — ①

\[ I = \frac{V}{R} = \frac{V_m \sin \omega t}{R} = \frac{V_m}{R} \sin \omega t \]

\( I = I_m \sin \omega t \) — ②
phasor diagram

\[ \phi = 0 \rightarrow VR \]

Power factor

\[ \cos \phi = \cos 0^\circ = 1 \text{ (UPF)} \]

\[ P_{avg} = \frac{1}{T} \int_0^T v(t) i(t) \, dt \]

\[ P_{avg} = V_{Im} \sin \omega t \times I_{Im} \sin \omega t \]

\[ = V_{Im} I_{Im} \left( \frac{1 - \cos 2\omega t}{2} \right) \]

\[ = \frac{V_{Im} I_{Im}}{2} - \frac{V_{Im} I_{Im} \cos 2\omega t}{2} \]

\[ P_{avg} = \frac{V_{Im}}{\sqrt{2}} \times \frac{I_{Im}}{\sqrt{2}} \]

\[ \text{P}_{avg} = V_{RMS} \cdot I_{RMS} \text{ watts.} \]

Inductor:

\[ V = L \frac{d}{dt} (I_{Im} \sin \omega t) = \omega L I_{Im} \cos \omega t \]

\[ = \omega L I_{Im} \cos (\omega t + 90^\circ) \]

\[ = \omega L I_{Im} \sin (\omega t) [j] \]

\[ V = j \omega L \overline{I} \]

\[ \overline{V} = +j \times L \overline{I} \]

Vltg is "j operator" times the current.

i.e. \( V \) leads \( I \) by \( 90^\circ \).
Where \( X_L = \omega L = 2\pi f L \)

Inductive reactance.

**Phasor diag**

Power factor

\[ PF = \cos \phi = \cos 90^\circ = 0. \]

**Capacitor**

Let \( \bar{V} = V_m \sin \omega t \)

\[ I = C \frac{d\bar{V}}{dt} \]

\[ I = C \frac{d(V_m \sin \omega t)}{dt} = C \cdot \omega V_m \cos \omega t \]

\[ = \omega C V_m \sin (\omega t + 90^\circ) \]

\[ = \omega C V_m \sin \omega t [j] \]

\[ I = j \omega C V \]
\[
\vec{V} = \frac{\vec{I}}{j\omega C} = -\frac{j}{\omega C} \vec{I}
\]

\[
\vec{V} = -jX_C \vec{I}
\]

where,

\[
X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}
\]

\[\text{Capacitive reactance}\]

\[\text{Phasor diag}\]

\[\text{Power factor}\]

\[p.f. = \cos \phi = \cos 90^\circ = 0\]
**Series R-L**

\[ \begin{align*}
\text{KVL} & : -V + VR + VL = 0 \\
V & = IR + jXL I \\
\frac{V}{I} & = Z = R + jXL \\
V & = IZ
\end{align*} \]

**Impedance \( Z \)**

\[ \begin{align*}
|Z| & = \sqrt{R^2 + X_L^2} \\
\phi & = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{\omega L}{R}\right)
\end{align*} \]

**Power factor**

\[ \begin{align*}
\cos\phi & = \frac{R}{|Z|} \\
\sin\phi & = \frac{X_L}{|Z|}
\end{align*} \]

**Power \( P \)**

\[ \begin{align*}
I^2 R + jXL I^2 & = P + jQ_L \\
VI & = I^2 R + jI^2 X_L
\end{align*} \]

\[ S = P + jQ_L \]

**In AC ckt, all the convertible part of energy is represented by Resistance**

- Electricity ➔ Heat
- Electricity ➔ Sound

**Active/Real Power (watts)**

**Total/Apparent Power (VA's)**

\[ \begin{align*}
|S| & = \sqrt{P^2 + Q_L^2} \\
\phi & = \tan^{-1}\left(\frac{Q_L}{P}\right)
\end{align*} \]
\[
\cos \phi = \frac{P}{S} \implies P = \cos \phi \times S = VI \cos \phi \text{ watts}
\]

\[
\sin \phi = \frac{Q_L}{S} \implies Q_L = S \sin \phi = VI \sin \phi \text{ var's}
\]

Here \( V, I \rightarrow \text{rms value} \)

**Phasor diagram:**

\( I \) lags \( V \) by \( \phi < 90^\circ \)

**Series RC**

\[
\begin{align*}
I & = \text{KV} \\
V_R & = R \rightarrow R \\
V_c & = \frac{1}{jX_c} C \rightarrow -jX_c \\
\end{align*}
\]

Here,

\[
Z = R - jX_c
\]

**Impedance \( \Delta I_e \):**

\[
|Z| = \sqrt{R^2 + X_c^2}
\]

\[
\phi = \tan^{-1} \left( \frac{X_c}{R} \right) = \tan^{-1} \left( \frac{1}{\omega RC} \right)
\]

**Impedance of circuit**

**Power factor:**

\[
\cos \phi = \frac{R}{|Z|} \quad \sin \phi = \frac{X_c}{|Z|}
\]
\[ I^2 = I^2 R - j X_c I^2 \]
\[ V I = I^2 R - j I^2 X_c \]

\[ S = P - j Q_c \]

\[ \text{Power angle} \]

\[ |S| = \sqrt{P^2 + Q_c^2} \]

\[ \phi = \tan^{-1} \left( \frac{Q_c}{P} \right) \]

\[ \cos \phi = \frac{P}{S} \Rightarrow P = S \cos \phi = V I \cos \phi \text{ (watts)} \]

\[ \sin \phi = \frac{Q_c}{S} \Rightarrow Q_c = S \sin \phi = V I \sin \phi \text{ (VAR's)} \]

\( V, I \rightarrow \text{rms value} \)

\[ \text{Phasor diag:} \]

\[ I \text{ leads } V \text{ by } \phi < 90^\circ \]

\( \oplus Q_L \rightarrow \text{absorbing VAR's} \text{ (lagging VAR's)} \)

\( \ominus Q_c \rightarrow \text{generating VAR's} \text{ (leading VAR's)} \)
**R-L-C Circuit**

\[ \mathbf{KVL} \quad \vec{V} = \vec{V}_R + \vec{V}_L + \vec{V}_C = 0 \]

\[ \vec{V} = I \left[ R + j \left( X_L - X_C \right) \right] \]

\[ V = I \, Z \]

Net: \[ Z = R + j \left[ \frac{X_L - X_C}{R} \right] \]

\[ \phi = \tan^{-1} \left[ \frac{X_L - X_C}{R} \right] \]

Net impedance angle

**Power Factor**

\[ \cos \phi = \frac{R}{|Z|} \]

\[ \sin \phi = \frac{|X_L - X_C|}{|Z|} \]

**Case 1** \( X_L > X_C \Rightarrow \text{(General nature of electrical sys.)} \)

\[ Z = R + j \, X_{net} \]

\[ I \text{ lags } V \text{ by } \phi < 90^\circ \text{ (lagging PF)} \]

**Case 2** \( X_L < X_C \)

\[ Z = R - j \, X_{net} \]

\[ I \text{ leads } V \text{ by } \phi < 90^\circ \text{ (leading PF)} \]

**Case 3** \( X_L = X_C \)

\[ Z = R \Rightarrow \text{purely resistive} \]

\[ I \text{ in phase with } V \Rightarrow \phi = 0^\circ, \text{ P.F.} = 1 \text{ (UPF)} \]
\[ Z = R \mp jX \]

\[ Z = R + jX_L \quad Z = R - jX_C \]

\[ \frac{1}{Z} \]

Eq:

\[ Z = 4 + j3 \]

\[ Y = \frac{1}{Z} \rightarrow \text{admittance (} W) \text{ or } S \]

\[ Y = \frac{1}{4 + j3} \times \frac{4 - j3}{4 - j3} = \frac{4 - j3}{25} \]

\[ Y = (0.16 - j0.12) \, W \]

\[ Y = G_r \pm jB \]

\[ Y = G_r - jB_L \]

where \[ B_L = \frac{1}{X_L} = \frac{1}{\omega L} = \frac{1}{2\pi fL} \]

L \rightarrow \text{inductive susceptance (} W) \]

\[ Y = G_r + jB_C \]

where \[ B_C = \frac{1}{X_C} = \frac{1}{1/\omega C} = \frac{1}{\omega C} = 2\pi fC \]

L \rightarrow \text{capacitive susceptance (} W) \]

impedance \rightarrow \text{admittance}

reactance \rightarrow \text{susceptance}
Phasor diagrams:

(a) Series R-L circuit:
\[
V_R = IR \angle 0^\circ \\
V_L = jX_L I \\
\cos \phi = \frac{V_R}{V} \text{ (lagging)}
\]

(b) Series R-C circuit:
\[
V_R = IR \angle 0^\circ \\
V_C = -jX_C I \\
\cos \phi = \frac{V_R}{V} \text{ (leading)}
\]

(c) Series R-L-C circuit:
\[
V_R = IR \angle 0^\circ \\
V_L = jX_L I \angle 90^\circ \\
V_C = jX_C I \angle -90^\circ \\
\text{Case 1: } \phi = \tan^{-1} \left( \frac{V_L-V_C}{V_R} \right) \\
\cos \phi = \frac{V_R}{V} \text{ (lagging)}
\]
\[ \text{Case 2: } X_L < X_C \Rightarrow V_L < V_C \]

\[ |V_L| = \sqrt{V_R^2 + (V_C - V_L)^2} \]
\[ \phi = \tan^{-1}\left(\frac{V_C - V_L}{V_R}\right) \]
\[ \cos \phi = \frac{V_R}{V} \text{ (leading)} \]

\[ \text{Case 3: } X_L = X_C \Rightarrow V_L = V_C \]

\[ |V_L| = V_R \]
\[ \phi = 0^\circ \]
\[ \text{P.F.} = \cos \phi = \cos 0^\circ = 1 \text{ (UPF)} \]

\[ (2) \text{ Parallel Ckts} \Rightarrow V \rightarrow \text{ref} \]

\[ \text{(a) Parallel R-L} \]

\[ \begin{align*}
I_R &= \frac{V}{R} \angle 0^\circ \\
I_L &= \frac{V}{jX_L} = \frac{V}{X_L} \angle 90^\circ
\end{align*} \]

\[ \text{(b) Parallel R-C} \]

\[ \begin{align*}
I_R &= \frac{V}{R} \angle 0^\circ \\
I_C &= \frac{V}{-jX_C} = \frac{V}{X_C} \angle 90^\circ
\end{align*} \]
(c) Parallel R-L-C

\[ I_R = \frac{V}{R} \cos \theta \]
\[ I_L = \frac{V}{X_L} \cos (\theta - \phi) \]
\[ I_C = \frac{V}{X_C} \cos \phi \]

**Case (1):** \( X_L > X_C \Rightarrow I_L < I_C \)

\[ |I| = \sqrt{I_R^2 + (I_C - I_L)^2} \]
\[ \phi = \tan^{-1} \left( \frac{I_C - I_L}{I_R} \right) \]
\[ \cos \phi = \frac{I_R}{I} \quad \text{(leading)} \]

**Case (2):** \( X_L < X_C \Rightarrow I_L > I_C \)

**Case (3):** \( X_L = X_C \Rightarrow I_L = I_C \)

\[ |I| = I_R \]
\[ \phi = 0^\circ \]
\[ \cos \phi = \cos 0^\circ = 1 \quad \text{(UPF)} \]
1) If \( A_1 \to \text{reads} \to 8\) A
   \( A_2 \to \text{reads} \to 6\) A
   \( A \to \text{reads} = \) __________
   
   \( \text{ckt P.F.} = \) __________

   \[ I = \sqrt{I_1^2 + I_2^2} = \sqrt{64 + 36} = \sqrt{100} = 10 \]

   \( \text{P.F.} = \frac{I_1}{I} = \frac{8}{10} = 0.8 \) (lagging)

2) If \( A_1 \to \text{reads} \to 6\) A
   \( A_2 \to \text{reads} \to 18\) A
   \( A_3 \to \text{reads} \to 10\) A
   \( A \to \text{reads} = \) __________
   
   \( \text{ckt P.F.} = \) __________

   \[ I = \sqrt{I_1^2 + (I_2 - I_3)^2} \]
   \[ = \sqrt{36 + 64} \]
   \[ = \sqrt{100} = 10 \]

   \( \text{P.F.} = \frac{I_1}{I} = \frac{16}{10} \]
   \[ = 0.6 \] (leading)

3) \( I = \) __________
   
   \[ \text{Find } I = \] __________

   \( \frac{200V}{20\Omega} + \frac{200V}{800} \)

   \( \text{ckt } \text{PF} = \) __________
RMS value / True / Effective value:

It is the steady value of a time-varying voltage or current waveform which could produce the same amount of heat as given by the original waveform for a definite period of time.

\[ V_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T [v(t)]^2 \, dt} \]

Average value / Mean value:

It is the steady value of equivalent voltage or current waveform which could develop the same amount of charge as given by the original waveform for a definite period of time in a circuit.
Concept of Symmetry:

1A1 = 1A21 \Rightarrow Symmetrical
1A1 \neq 1A21 \Rightarrow Asymmetrical

The average value of any symmetrical waveform for one full cycle is always zero.

(a) For symmetrical waveform:

\[ \text{Vavg} = \begin{cases} 0 & \text{full cycle} \\ \frac{1}{T/2} \int_{T/2}^{T} v(t) \, dt & \text{half cycle} \end{cases} \]

(b) For asymmetrical waveform:

\[ \text{Vavg} = \frac{1}{T} \int_{0}^{T} v(t) \, dt \rightarrow \text{full cycle} \]
Peak factor / Crest Factor:

\[ \frac{V_{\text{max}}}{V_{\text{rms}}} \]

Form Factor / Shape Factor:

\[ \frac{V_{\text{rms}}}{V_{\text{avg}}} \]

Peak to Peak value:

\[ V_{p-p} = |V_{\text{max}} - V_{\text{min}}| \]

Note:

Most of our electrical utilities applications involve heat generation so we talk RMS values in general.

Eg:- 1Ø, Domestic supply in India = 230 V → RMS value

However applications like battery charging, electroplating, electro refining process, etc involves charge, so we calculate “avg” values.

Standard waveforms:

\[ V_{\text{rms}} = k \quad F.F. = 1 \]
\[ V_{\text{avg}} = k \quad V_{pp} = k \]
\[ \text{P.F.} = 1 \]
\[ V_{\text{RMS}} = \frac{V_m}{\sqrt{2}} \]

For a sawtooth wave, \( V_{\text{RMS}} = \frac{V_m}{\sqrt{2}} \)

\[ V_{\text{avg}} = \frac{2V_m}{\pi} \rightarrow \text{Half cycle} \]

\[ P.F. = \sqrt{2} \quad F.F. = \frac{\pi}{2\sqrt{2}} = 1.11 \quad V_{pp} = 2V_m \]

\[ V_{\text{RMS}} = \frac{V_m}{\sqrt{2}} \]

\[ V_{\text{avg}} = \frac{2V_m}{\pi} \]

\[ P.F. = \sqrt{2} \quad F.F. = 1.11 \quad V_{pp} = V_m \]

\[ V_{\text{RMS}} = k \]

\[ V_{\text{avg}} = 0 \rightarrow \text{Full cycle} \]

\[ v_{\text{avg}} = k \rightarrow \text{Half cycle} \]

\[ P.F. = 1 \quad F.F. = 1.57 \quad V_{pp} = V_m \]

\[ f_{\text{RMS}} = A\sqrt{\frac{2c}{y}} \]

\[ f_{\text{avg}} = A\left[\frac{2c}{y}\right] \]
$I_{RMS} = \frac{I_0}{\sqrt{2}}$  \hspace{1cm}  $I_{avg} = \frac{I_0}{2}$

$P.F = \sqrt{2}$  \hspace{1cm}  $F.F = \sqrt{2}$

$I_{pp} = I_0$

$V_{RMS} = \frac{k}{\sqrt{2}}$  \hspace{1cm}  $V_{avg} = \frac{k}{2}$

$P.F = \sqrt{3}$  \hspace{1cm}  $F.F = \frac{2}{\sqrt{3}}$

$V_{pp} = k$

$I_{RMS} = \frac{A}{\sqrt{3}}$

$I_{avg} = 0 \rightarrow$ full cycle

$= \frac{A}{2} \rightarrow$ half cycle

$P.F = \sqrt{3}$  \hspace{1cm}  $F.F = \frac{2}{\sqrt{3}}$

$V_{pp} = 2A$

$V_{RMS} = \frac{k}{\sqrt{3}}$  \hspace{1cm}  $V_{avg} = \frac{k}{2}$

$P.F = \sqrt{3}$  \hspace{1cm}  $F.F = \frac{2}{\sqrt{3}}$

$V_{pp} = 2A$
**Note:**

**AC Analog Meters**
- Moving Iron type
- RMS values

**DC Analog Meters**
- PMMC Type
- Average value

**Rectifier Type**

\[
\begin{align*}
\text{final value} &= \text{Avg. value} \times F.F. \\
&= \text{final value} \times 0.95 \text{ (of sinusoidal waveform)}
\end{align*}
\]

→ Practical waveforms are not std.

So, we use Fourier series expansion to express these practical **voltage & current waveforms** in terms of sine (or) cosine.

\[ v(t) = V_0 + V_1 \sin \omega t + V_2 \sin 2\omega t + \ldots \]  

\[ V_{avg} = V_0 \]

\[ V_{rms} = \sqrt{V_0^2 + \left(\frac{V_1}{\sqrt{2}}\right)^2 + \left(\frac{V_2}{\sqrt{2}}\right)^2 + \ldots} \]
\[ i(t) = I_0 + I_1 \cos(\omega t - \phi_1) + I_2 \cos(3\omega t + \phi_3) + \ldots \]

\[ I_{\text{avg}} = I_0 \]

\[ I_{\text{RMS}} = \sqrt{I_0^2 + \left(\frac{I_1}{\sqrt{2}}\right)^2 + \left(\frac{I_2}{\sqrt{2}}\right)^2 + \ldots} \]

**Non sinusoidal excitation:**

\[ \hat{i}(t) + \hat{v}(t) = V_0 + V_1 \sin \omega t + V_3 \sin 3\omega t + V_5 \sin 5\omega t + V_7 \sin 7\omega t \]

\[ i(t) = I_0 + I_1 \sin (\omega t - \phi_1) + I_3 \sin (3\omega t + \phi_3) + I_5 \cos (5\omega t - \phi_5) + I_7 \sin (7\omega t + \phi_7) \]

What is avg. power in \( n/\omega \)?

**Note:**

If freq. are same, don't use Fourier series concept.

\[ v(t) = v_1 \sin (\omega t + \phi) + v_2 \sin (\omega t - \phi) \]

\[ = v_1 \angle \phi + v_2 \angle -\phi \]

**Phasor addition**

\[ \Rightarrow V_{\text{RMS}} = \frac{V_{\text{max}}}{\sqrt{2}} \]

**Soh**

\[ P_{\text{avg}} = \frac{1}{T} \int_0^T v(t) \cdot i(t) \, dt \]

\[ P_{\text{avg}} = V_0 I_0 + \frac{V_1}{\sqrt{2}} \cdot I_1 \cos \phi_1 + \frac{V_3}{\sqrt{2}} \cdot I_3 \cos \phi_3 + \ldots + \frac{V_5}{\sqrt{2}} \cdot I_5 \cos (90 - \phi_5) \]
Active power is one value of as power waveform by itself the product of voltage & current waveform.

This active power exist in resistive part of the load which is the consumable / convertible part.

So, simply active power means active power in watts.

\[ P = V_{\text{rms}} \cdot I_{\text{rms}} \cos \phi \]

\[ v(t) = 10 + 5\sqrt{2} \cos \omega t + 3\sqrt{2} \sin 3\omega t \]

\[ v(t) = 10 + 5\sqrt{2} \sin (90^\circ + \omega t) + 3\sqrt{2} \sin 3\omega t \]

\[ V_{\text{rms}} = \sqrt{10^2 + (5\sqrt{2})^2 + (3\sqrt{2})^2} \]

\[ = \sqrt{134} \approx 11.57 \]

\[ V_{\text{avg}} = 10 \]
3) \( i(t) = 10 + 7 \cos (\omega t - 10^\circ) + 5 \cos (3\omega t + 30^\circ) \)

\[
I_{avg} = 10 \\
I_{rms} = \left( (10)^2 + (\frac{7}{\sqrt{2}})^2 + (\frac{5}{\sqrt{2}})^2 \right)^{1/2} = \sqrt{137} = 11.7
\]

\[i(t) = 20 \cos (\omega t - 30^\circ) + 7 \cos (\omega t + 40^\circ)\]

\[= 20 \cos (\omega t - 30^\circ) + 7 \sin (\omega t + 130^\circ)\]

\[
\sqrt{20^2 + (7 \sqrt{2})^2} = 20 + 8.66 - 5^j + (-4.5 + 5.36j) = 24.16 + 0.36j
\]

\[= 20 + \left[ 2.17 \angle 4.9^\circ \right] \]

\[
I_{rms} = \sqrt{(20)^2 + (\frac{2.17}{\sqrt{2}})^2} = 20.21 \quad I_{avg} = 20
\]

5) \[\frac{3\sqrt{2}}{\tan \theta} \]

\[
p_{lost} = I_{rms} \times R = (\frac{3\sqrt{2}}{3})^2 \times 3 = 25W
\]

6) \[\frac{3\sqrt{2}}{\tan \theta} \]

\[
p_{lost} = \left[ \frac{V_{rms}^2}{R} \right]
\]
\[ V_{RMS} = \sqrt{\frac{1}{4} \left[ \int_0^2 (8t)^2 \, dt + \int_4^8 (16)^2 \, dt \right]} \]

\[ = \sqrt{\frac{1}{4} \left[ \frac{25}{3} \left( \frac{3}{13} \right) + 100 \times 2 \right]} = \sqrt{66.66} \]

\[ P_{lost} = \frac{(V_{RMS})^2}{2} = \frac{66.66}{2} = 33.33 \text{ W} \]

7) Find \( V_{avg}, V_{RMS}, P.F., F.F., V_{pp} \)

\[ V(t) \]

\[ \begin{array}{c}
0 < t < 2 \\
V(t) = 8t \\
\frac{4 \pi + 2\pi}{8}
\end{array} \quad \begin{array}{c}
4 \leq t < 8 \\
V(t) = 16
\end{array} \]

\[ V_{avg} = \frac{1}{8} \left[ \int_0^2 (8t)^2 \, dt + \int_2^4 16 \, dt + \int_4^8 (-4t+32)^2 \, dt \right] \]

\[ = \frac{1}{8} \left[ 4(2)^3 + 16(2) + 32(4) - 2(48) \right] = 10 \text{ V} \]

\[ V_{RMS} = \sqrt{\frac{1}{8} \left[ \int_0^2 (8t)^2 \, dt + \int_2^4 (16)^2 \, dt + \int_4^8 (-4t+32)^2 \, dt \right]} \]

\[ = \sqrt{\frac{1}{8} \left[ \frac{64(8)}{3} + (16)^2(2) + (32)^2(4) + \frac{1448}{2} - 128(48) \right]} \]

\[ = 8\sqrt{2} \quad P.F. = \frac{16}{8\sqrt{2}} = \sqrt{2} \quad F.F. = \frac{8\sqrt{2}}{10} = 1.13 \]
8. Total power absorbed by cell = \underline{9}

\[ P_{abs} = (i_{avg})^2 R + (i_{avg})^2 E \]

\[ i_{avg} = \frac{4}{2} = 2 \]

\[ P_{abs} = \left( \frac{4}{\sqrt{3}} \right)^2 \times 1 + (2) \times 5 \]

\[ = \frac{16}{3} + 10 = \frac{46}{3} \]

\[ = 15.33 \text{ W} \]

9) If a current of \( i(t) = 5 \cos(1000t + 100^\circ) \) A is flowing through an impedance of \( (4 + j3) \) \( \Omega \), the avg. power is –?

\[ i(t) = 5 \cos(1000t + 100^\circ) \]

\[ z = 4 + j3 \quad \Rightarrow \quad R = 4 \]

\[ X_L = 3 \]

\[ P_{avg} = \frac{1}{T} \int_0^T v(t) \cdot i(t) \, dt \]

\[ \rightarrow \text{exist only for resistive element} \]

\[ \rightarrow \text{active power} \]

\[ P_{avg} = (i_{rms})^2 \times R \]

\[ = \left( \frac{5}{\sqrt{2}} \right)^2 \times 4 \]

\[ = \frac{25}{2} \times 4 \]

\[ = 50 \text{ W} \]
If the periodic 60 Hz waveform shown is given to a PMMC type voltmeter then what is its reading?

PMMC → Avg Value

\[ \text{Avg} = \frac{1}{T} \int_{0}^{T} v(t) \, dt \text{ which is area under the curve } v(t) \]

\[ \text{Avg} = \left\{ \frac{1}{2} \times 25 \times 2 + 2 \times 25 + \frac{1}{2} \times 2 \times 35 + 2 \times 25 \right\} - \left\{ \frac{1}{2} \times 2 \times 10 + 2 \times 10 \right\} \]

\[ \text{Avg} = \text{Net area} = 250 - 30 = 220 \]

\[ \text{Avg} = \frac{\text{Area}}{T} = \frac{220}{4} = 55 \, V \]

Find \( E_{\text{rms}}, \, R_{\text{ms}}, \, \text{Avg, P.F., FF, Vpp} \).
1) If \( \omega = 4 \text{ rad/sec} \), find \( \eta / \omega \) P.F.

\[
Z_{eq} = \frac{2 + (4+j4)}{1} \left( -2j \right)
\]

\[
= \frac{2 - j8}{\sqrt{(2-8)^2 + (2j)^2}}
\]

\[
\cos \phi = \frac{R}{1Z1} = \frac{2 - j8}{\sqrt{(2-8)^2 + (2j)^2}}
\]

\[
\therefore \text{P.F.} = 0.76 \text{ (leading)}
\]

2) \[ (2+j2)\Omega \]

\[ (1+j2)\Omega \]

\[ (1-j2)\Omega \]

\[ (1+j2)\Omega \]

\[ Z_{eq} = \left[ (2+j2) + \frac{(1-j2)}{(1+j2)} \right] \]

\[ = 2 + j2 + 1 \]

\[ = 3 + j2 \]

3) \[ (2+j2)\Omega \]

\[ 4\Omega \]

\[ -j2\Omega \]

\[ b \]

\[ a \]

\[ c \]

\[ Z_{eq} = 0 \]

4) \[ (2+j2)\Omega \]

\[ 2\Omega \]

\[ -j2\Omega \]

\[ (1+j1)\Omega \]
5) \( f \omega = 10 \text{ k rad/sec} \), \( Z_{in} = \)

\[ Z_{in} = 1 \mu F \]

\[ V_x = 50(1) = 50 \text{ V} \]

\[ V_T = 150 + j100 \]

\[ Z_{in} = \frac{V_t}{120} = (150 + j100) \Omega \]

6) \( i(t) \)

\[ V(t) = 20 \cos (100t - 40^\circ) \]

\[ i(t) = 8 \sin (100t + 10^\circ) \]

Determine \( n/w \) elements.

Here,

\[ V(t) = 20 \cos (100t - 40^\circ) \Rightarrow 20 \angle 50^\circ \]

\[ i(t) = 8 \sin (100t + 10^\circ) \Rightarrow 8 \angle 10^\circ \]

\[ Z = \frac{V}{I} = \frac{20 \angle 50^\circ}{8 \angle 10^\circ} = 2.5 \angle 40^\circ \]

Simple form of \( n/w \) could be R-L series circuit

\[ R = 1091 \Omega \]

\[ X_L = 1.6 \Rightarrow \omega L = 1.6 \Rightarrow L = 16 \text{ mH} \]

It can also be a R-L-C n/w when \( R \times X_L > X_C \)
7) Find $V_o = \ldots$

Here $V = 20 \cos zt$
\[ \omega = 2 \text{ rad/ sec} \]
\[ V_{rms} = 20 \text{ V} \]

\[ V_o = \frac{20 \cos \theta}{\sqrt{2}} \frac{1}{1 + j^2} \]

\[ \therefore \quad V_o = \frac{(10\sqrt{2} \angle 0 \text{ V}) (1)}{(1+j^2)} \]

\[ V_o = (11.3 + 5.6 j) \text{ V} \]

8) Find $I_o = \ldots$

\[ I_o = \frac{(20 \angle 0 \text{ A}) (4 + j3)}{4 + j1} \]

\[ = \ldots \]

9) \[ Z = 4 + j3 - (j3) = 4 + j3 \]
\[ |Z| = 5 \text{ ohms} \]
\[ \gamma = \frac{1}{4 + j3} = \frac{4 - j3}{25} \quad \text{or} \quad (0.16 - j0.12) \text{ ohms} \]
\[ |\gamma| = \frac{1}{|Z|} = 0.2 \text{ S} \]

In phasor representation of polar quantity, \( \gamma \angle 0 \), \( \gamma \) is clearly representing the max. value.

Find: 1) $Z$ 2) $Y$ 3) $I$

4) $V_R$, $V_L$, $V_C$

5) Verify KVL

6) $P$, $Q$, $S$

7) Phasor diag.
\[ I = \frac{30}{\sqrt{2}} \angle 30^\circ \quad V = \frac{30}{\sqrt{2}} \angle 36.86^\circ \]

\[ R_{\text{US}} = 4 \angle 36.86^\circ \quad \Rightarrow \text{impedance angle} \]

\[ I_R = I_l = I_c \]

\[ V_R = I_R \cdot R = (4.24 \angle -36.86^\circ) \cdot 4 \]

\[ = 16.96 \angle -36.86^\circ \quad \text{V} \]

\[ V_L = I_L (jX_L) = (4.24 \angle -36.86^\circ) (j5) \]

\[ = 21.44 \angle 53.14^\circ \quad \text{V} \]

\[ V_C = I_C (\bar{j}X_C) = (4.24 \angle -36.86^\circ) (-j3) \]

\[ = 4.24 \angle -36.86^\circ 12.72 \angle -126.86^\circ \quad \text{V} \]

\[ \bar{V}_S = \bar{V}_R + \bar{V}_L + \bar{V}_C \quad \text{[KVL \rightarrow RMS value]} \]

\[ R_{\text{US}} \quad l\bar{V}_R + l\bar{V}_L + l\bar{V}_C \]

\[ = 16.96 + 21.44 + 12.72 = 21.16 \]

\[ l\bar{V}_S = \frac{30}{\sqrt{2}} = 15\sqrt{2} = 21.2 \]

\[ \therefore \text{LHS} = \text{RHS} \]

\[ \therefore \text{KVL verified.} \]

\[ \text{For calculating power consider RHS mag. only} \]

\[ P = I_R^2 R = \frac{V_R^2}{R} = V_R \cdot I_R \]

\[ = (16.96)^2 (4.42)^2 4 = \frac{(16.96)^2}{4} = (16.96)(4.42) \]

\[ = 71.91 \text{ \text{Watt}} \]

\[ \Theta_{\text{net}} = \Theta_L - \Theta_C \]
\[ \Theta_L = I_L^2 \cdot X_L = \frac{V_L^2}{X_L} = V_L \cdot I_L. \]
\[ = (4.42^2 \cdot 6) = \left(\frac{25.44}{6}\right)^2 = (25.44)(4.42) \]
\[ = 107.86 \text{ VAR (absorbing)} \]
\[ \Theta_c = \frac{I_c^2}{X_c} = \frac{V_c^2}{X_c} = V_c \cdot I_c \]
\[ = (4.42^2 \cdot 3) = \left(\frac{12.72}{3}\right)^2 = (12.72)(4.42) \]
\[ = 53.93 \text{ VAR's (generating)} \]
\[ \Theta_{net} = 107.86 - 53.93 \]
\[ = +53.93 \text{ VAR's (absorbing / lagging)} \]

2. Total Power
\[ S = V_s \cdot I_s = 1 I_s^2 Z = \frac{1 V_s^2}{Z} \]
\[ = (21.21)(4.24) = (4.04)^2(5) = \left(\frac{21.21}{5}\right)^2 \]
\[ = 89.93 \text{ VA's} \]

3. Power Factor
\[ \cos \Phi = \frac{R}{Z} = \frac{P}{S} \]
\[ = \cos (36.36) = \frac{4}{5} = \frac{71.74}{89.93} \]
\[ = 0.48 \text{ (lagging)} \]

Also check:
\[ |Z| = \sqrt{R^2 + (X_L - X_c)^2} \]
\[ \Phi = \tan^{-1} \left( \frac{X_L - X_c}{R} \right) \]
\[ |S| = \sqrt{P^2 + (\Theta_L - \Theta_c)^2} \]
\[ \Phi = \tan^{-1} \left( \frac{\Theta_L - \Theta_c}{P} \right) \]
\[ P = S \cos \Phi = V_s I_s \cos \Phi \]
\[ \Theta_{net} = S \sin \Phi = V_s I_s \cos \Phi \]
\[ V_s = 21.21\ \angle 0^\circ \]
\[ I_s = I_R = I_L = I_C = 4.42\ \angle -36.86^\circ \ \text{A} \]
\[ V_R = 16.96\ \angle -36.86^\circ \ \text{V} \]
\[ V_L = 25.44\ \angle 53.14^\circ \ \text{V} \]
\[ V_C = 12.72\ \angle -126.86^\circ \ \text{V} \]

Normally we take \( V_s \) as ref. but here we take \( V_s \) as ref., becoz all calculations are done w.r.t. \( V_s \).

10) Find \( V_{ac} \) using mesh & nodal analysis.

**Mesh**

\[-[10\angle 0^\circ] + 1[I_1 - I_3] + j1[I_1 - I_2] = 0\]

\[(1+j)I_1 - jI_2 - I_3 = 0\]  \[\text{L} \]

\[ I_2 = -[5\angle 30^\circ]\]  \[\text{L} \]

\[ I_3 = \frac{1}{2} [I_2 - I_3] + [I_3 - I_1] = 0\]

\[-I_1 + jI_2 + (3-j)I_3 = 0\]  \[\text{3}\]

\[(1+j)I_1 - I_3 = 10 - 5\angle 120^\circ\]  \[\text{A}\]

\[ I_1 = \frac{12.5 - j4.33}{1}\]

\[-I_1 + (3-j)I_3 = 5\angle 120^\circ\]  \[\text{B}\]

\[-2.5 + j4.33\]
(A) \[ (1+j) I_1 - I_3 = 12.5 - j4.83 \]

(B) \[ (1+j) I_1 + (1+j) I_3 = -6.83 + j1.83 \]

\[ (3+j2) I_3 = 5.67 - j2.5 \]

\[ I_3 = 1.71 \angle 57.48^\circ \ A \]

Then, \[ V_{nc} = 2 I_3 = 3.42 \angle -57.48^\circ \ V \]

**Node 1**

\[ V_1 = 10 \text{ V} \]

\[ \frac{V_2 - 10}{1} + \frac{V_2}{j^1} + \frac{V_2 - V_3}{-j^1} = 0 \]

\[ V_2 - 10 - jV_2 + j(V_2 - V_3) = 0 \]

\[ V_2 - jV_3 = 10 \text{ V} \]

\[ - \left[ 5 \angle 30^\circ \right] + \frac{V_3 - V_2}{-j^1} + \frac{V_3 - 10}{2} = 0 \]

\[ \frac{j(V_3 - V_2)}{1} + \frac{(V_3 - 10)}{2} = 5 \angle 30^\circ \]

\[ -j2 V_2 + (1 + j2) V_3 = 10 + 10 \angle 30^\circ \]

\[ = 18.66 + j5 \text{ V} \]

\[ \times j2 \Rightarrow j2 V_2 + 2 V_3 = j20 \]

\[ -j2 V_2 + (1 + j2) V_3 = 18.66 + j5 \]

\[ V_3 = 8.65 \angle 19.57^\circ \]

\[ V_{nc} = \bar{V}_1 - \bar{V}_3 = 10 - [8.66 \angle 19.57^\circ] \]

\[ = 3.43 \angle -57.44^\circ \ V \]
Source Transformation Technique:

Here \( I = 2 \angle 45^\circ \); \( \omega = 2 \)

\[ \overline{Z} = (1 + j4) - j1 = (1 + j3) \Omega \]

\[ \overline{V} = \overline{I} \cdot \overline{Z} = 2 \angle 45^\circ \cdot (1 + j3) \]

\[ = 2 \angle 45^\circ \times 3.16 \angle 71.56^\circ \]

\[ = 6.32 \angle 116.56^\circ \quad \overline{V} \]

\[ \therefore \overline{V} = 6.32 \cos [2t + 116.56] \quad \overline{V} \]

Theorem 2: Superposition theorem.
Theorem 3: Thevenin's theorem.
Theorem 4: Norton's theorem.

(1) Superposition:

1. 10 V only: \( i = \frac{10 L_0}{5 + j2} = \frac{10 L_0}{5.38} \angle 28.8^\circ \)
\[ i' = 1.857 \angle -21.8^\circ \]

\[ 5 \text{A only; } \]

\[ i'' = 5\angle 30^\circ \left( \frac{2 + j2}{5 + j2} \right) = 5\angle 30^\circ \times (0.482 + 0.24j) \]

\[ = 2.61 \angle 53.19^\circ \]

\[ i = i' + i'' = 3.58 \angle 23.2^\circ \text{ A} \]

\( \{2\} \text{ Thevenin} \)

\[ Z_{TH} = (2 + j2) \Omega \]

\[ KVL \]

\[-[10 \angle 0^\circ] + V_{TH} - (2 + j2)(5\angle 30^\circ) = 0 \]

\[ V_{TH} = 19.32 \angle 45^\circ \]

\[ I = \frac{19.32 \angle 45^\circ}{5 + j2} = 3.58 \angle 23.2^\circ \text{ A} \]

\( \{3\} \text{ Norton's} \)

\[ I = \frac{10 \angle 0^\circ + 5\angle 30^\circ}{(2 + j2)} = \frac{10 \angle 0^\circ + 4.33 + 2.5j}{2.82 \angle 45^\circ} \]

\[ = 6.83 \angle 2.1^\circ \text{ A} \]
This circuit is multi-frequency excited. So in time domain only superposition theorem can provide the solution. But we can also do this problem in freq. domain by applying Laplace Transform.

\[ I = 6.83 \times 10 \left[ \frac{2+j2}{5+j2} \right] \]
\[ = 3.58 \times 23.2 \, \text{A} \]

1. **30V AC source only.**

\[ I' = \frac{30 \angle 0^\circ}{(1+j3)+[211+j]} \times \frac{2}{2+j1} \]
\[ = 6.63 \angle -96^\circ \, \text{A} \]

2. **10V DC source only**

\[ I'' = \frac{10}{2} = 5 \, \text{A} \]

By SPT

\[ I = 5 + 6.63 \cos (2t - 96^\circ) \, \text{A} \]
The minimal equivalent resistance across the load:

\[ V_L = 100V \angle 53.12^\circ \left( \frac{30^\circ}{3 + j401} \right) = 80 \angle 90^\circ \]

\[ V_{in} = 10V_L = 800 \angle 90^\circ \]

For the inverter:

\[ i(t) = \frac{10V_0}{1 + j1} = 7.67 \angle -45^\circ \]

By superposition:

\[ i(t) = 7.07 \sin(t-45^\circ) \]
\[ \sqrt{\left( \frac{10\angle20^\circ}{50\angle70^\circ} \right) \left( 30\angle40^\circ \right) } = \sqrt{6 \angle 10^\circ} = \sqrt{6} e^{-j10^\circ} \]

\[ 1 = \sqrt{6} e^{-j5^\circ} = \sqrt{6} \angle -5^\circ \]

**Theorem 5**

*Maximum Power Transfer Theorem.*

Though there are 3 types of physical power existing in AC steady state networks, the power that is consumable, utilizable, or convertible in any other form is *active power* or *real power* in watts.

So more, power transfer theorem is confined to active power only.

**General case:**

In this case both the resistive part and reactive part are controllable.

Since our target is to maximize active power in the load, \( (\Re \Re_e) \), here if we can compensate the net reactance, so total power = active power.

So \( P_{\text{max}} \) occurs in the load if load
impedance is complex conjugate of equivalent source impedance seen by it. i.e. \( Z_e = Z_s^* = R_s - jX_s \)

\[
\text{P}_{\text{max}} = \frac{|V_s|^2}{4R_s} \quad \text{W} \quad \text{(max value)}
\]

**In General:**

\( \text{P}_{\text{max}} \) occurs in the load when \( Z_e = Z_{\text{TH}}^* \)

and, \( \text{P}_{\text{max}} = \frac{|V_{\text{TH}}|^2}{2R_{\text{TH}}} \); \( R_{\text{TH}} = \text{Real part} \ [Z_{\text{TH}}^*] \)

**Special case:**

(a) Since load is purely resistive, let source have same reactance, which here we cannot cancel or real.

Since phase balancing of impedances is not possible so to extract more power, at least balance the magnitude so \( \text{P}_{\text{max}} \) occurs in the load.

\[
R_e = 1Z_{\text{TH}} = \sqrt{R_s^2 + X_s^2}
\]

Now to calculate \( \text{P}_{\text{max}} \), resubstitute the calculated value of \( R_e \) to find rms current in it. Then calculate

\[
\text{P}_{\text{max}} = I_{\text{rms}}^2 \times R_e
\]
Use Thevenin concept to all other general circuit.

**Special Case:** (b)

\[
Re = |R_s + jX_s + jX_e| = \sqrt{R_s^2 + (X_e + X_s)^2}
\]

**Special Case:** (c)

For \( P_{\text{max}} \):

\[
X_e = -X_s
\]

\[
X_e + X_s = 0
\]

Here, we can multiply the net reactance & reactive power but still, reactance cannot be controlled so \( P_{\text{max}} \) occurs in the load if \( X_e = -X_s \).

\[
\begin{align*}
\text{Value of } Z_e & \\
\text{for which } P_{\text{max}} \text{ occurs} & \\
\text{also find } P_{\text{max}}
\end{align*}
\]

\[ \rightarrow \text{Determine Thevenin equiv. across } Z_e. \]

1. \( Z_{TH} = (2 + j2) \, 5 \Omega \)
2. \( KVL \rightarrow [500^\circ] + [(2 + j2)(20e30^\circ)] + V_{TH} = 0 \)

\[ \therefore V_{TH} = 84.064 \angle 40.2^\circ \]
\[ Z_L = Z_{in}^* = (2 - j2) \Omega \]

\[ P_{\text{max}} = \frac{|V_{\text{TH}}|^2}{R_{\text{TH}}} = \frac{184.64^2}{2 \times 2} = 89.5 \text{ W} \]

\[ Z_{\text{TH}} = \frac{6+j8}{6+j8} = 3+j4 \Omega \]

What is \( P_{\text{max}} \) transferred to load?

1. \[ Z_{\text{TH}} = (6+j8) \div (6+j8) = (3+j4) \Omega \]

2. \( i = 180^\circ \). Nodal Analysis:

\[ \frac{V_{\text{TH}} - 110^\circ}{6+j8} + \frac{V_{\text{TH}} - 90^\circ}{6+j8} = 0 \]

\[ 2V_{\text{TH}} = 200 \angle 0^\circ \]

\[ V_{\text{TH}} = 100 \angle 0^\circ \]

\[ R_L = |Z_{\text{TH}}| = \sqrt{3^2 + 4^2} = 5 \Omega \]

\[ I_{5\Omega} = \frac{100^\circ}{(3+j4+5)} = 11.18 \angle 26.56^\circ \]

\[ P_{\text{max}} = 111.18^2 \times 5 = 625 \text{ W} \]
3) What is $P_{max}$ transferred to load?

\[ R = 14 + j4 - j11 = \sqrt{16 + 9} = 5 \Omega \]

\[ I_{RMS} = \frac{\frac{20}{\sqrt{2}} \angle 0^\circ}{(4 + j4 + 5 - j1)} = \frac{14 - 142 \angle 18.04^\circ}{9.486 \angle 18.43^\circ} \]

\[ = 1.49 \angle -18.43^\circ \]

\[ P_{max} = 1.49^2 \times 5 = 11.10 \text{ W} \]

4) What is the value of $R$ for which $P_{max}$ occurs with $R_{th}$?

\[ R_{th} = \frac{(2 - j2) \Omega}{(2 + j2) \Omega} = \frac{2}{4} = 0.5 \]

\[ R = 12 \Omega = 2 \Omega \]

5) $P_{max}$ transferred to load?

\[ Z_{th} = 10 + j4 \]

\[ V_{th} = 100 \angle 0^\circ \quad \text{Hm:} \quad X = -j4 \]
\[ P_{\text{max}} = I^2 R \]
\[ = \left( \frac{700 \times 20}{20} \right)^2 \times 10 = 250 \text{ W} \]

6) For what value of \( R \) is all power transferred from clkt \( A \) to clkt \( B \).

**Exact Proof**

\[ \frac{10 - 5}{2} = \frac{5 - 3}{R} \Rightarrow R = \frac{2 	imes 2}{5} = 0.8 \Omega \]

\[ I_L = \frac{40 - 3}{2 + R} = \frac{7}{2 + R} \]

\[ P_L = I_L^2 R = \frac{49}{(2 + R)^2} \times R + 3 \left( \frac{7}{2 + R} \right) \]

\[ = \frac{7R}{2 + R} + 3 \left( \frac{7R}{2 + R} + 3 \right) = \frac{70R + 42}{(2 + R)^2} \]

For \( P_{\text{max}} \):

\[ \frac{dP_L}{dR} = 0 \]
\[
\left[ \frac{(2+R)^2}(70) - (70R + 42) \cdot \left( \frac{2}{2+R} \right) }{(2+R)^2} \right] = 0
\]

\[
(2+R)^2(70) = 2(2+R)(70R + 42) \\
140 + 70R = 140R + 84 \\
70R = 56 \\
R = 0.852
\]

**Concept of Complex Power** \( S^* \)

\( S^* = V I^* \)

It is a mathematical concept which removes the confusion in calculating the 3 physical powers: S, P, Q in any new or element.

It will be very essential to calculate instantaneous power in AC circuits to estimate, design, control & analyze stability, fault calculations, etc in electrical power system since load is very dynamic in nature.

To verify Tellegen's theorem, to calculate power at any instant, we need complex power.

Hence, \( I^* \) complex conjugate \( I \)

\[ I = 10 \angle 20^\circ \text{ A} \Rightarrow I^* = 10 \angle -20^\circ \text{ A} \]
Units of $S^*$: Volt-Ampere (VA's).

We can also write:

$S^* = V^* I$

But generally we write:

$S^* = V I^*$

As $V \rightarrow$ is the cause & $I \rightarrow$ is the effect.

**Ex.1**

In one element

$V = 1V \angle \alpha$

$I = 1A \angle \beta$

$\rightarrow$ If we want to calculate instantaneous power

$V \cdot I = 1V \cdot 1A \angle \alpha + \beta \times$

$\rightarrow$ Correct way of calculating instantaneous power

$V \cdot I^* = 1V \cdot 1A \angle \alpha - \beta$

**Ex.2**

$V = 1V \angle \alpha$

$I = 1A \angle \beta$

$\rightarrow$ just $V \cdot I$ product

$1V \cdot 1A \angle \alpha - \beta \times$

$\rightarrow$ Consider $S^* = V I^*$

$= 1V \cdot 1A \angle \alpha + \beta$
1) \( n/w \) \( \begin{align*}
\text{Here, } V(t) &= 20 + j12 \\
i(t) &= 5 + j4
\end{align*} \)

Calculate: \( P, Q \) in the \( n/w \)

\( V(t) \rightarrow 20 + j12 \rightarrow 23.32 \angle 30.96^\circ \) V

\( i(t) \rightarrow 5 + j4 \rightarrow 6.4 \angle 38.65^\circ \) A

Complex Power \( S^* = V^* I^* \)

\[ S^* = 149.248 \angle -67.69^\circ \]

\[ S^* = 147.99 - 19.97 j \text{ VA} \]

For our \( n/w \):

\[ P = 147.9 \text{ watts} \]

\[ Q_c = 19.97 \text{ VAR's (generating)} \]

2) \( \begin{align*}
\text{If } V_R &= 5V, V_C &= 4 \sin 2t \ V, V_L = ? \\
2 + \frac{V_R}{5} &= \frac{dV_C}{dt} + \frac{1}{2} S v_I dt \\
\frac{1}{2} S v_C dt &= 2 + 4 - 8 \cos 2t \\
V_L &= \frac{d}{dt} (6 - 16 \cos 2t) \\
V_L &= 32 \sin 2t \ V
\end{align*} \)

3) \( i(t) = (4e^{-3t} + 3e^{-4t}) \ A \)

\( i_L(0) = 4 \ A \), \( \phi = \text{?} \)

\( \text{KCL: } i_L + 12 \cos (\omega t - \phi) = i_R \)

\( (\text{at } t = 9) \ i_L(0) + 12 \cos (-\phi) = 4 + 3 \)

\( \phi = 60^\circ \)
4) The power dissipated in 6Ω resistors is 0 W. Hence, what is V?

Here, \( P_{6Ω} = 0 \) W \( \Rightarrow \) I across 6Ω resistor is zero.

\( V_1 = V_2 \) (same in mag. & phase).

\[ V_1 = 20\angle0^\circ \left[ \frac{31\angle0^\circ}{1+31\angle0^\circ} \right] = 10\sqrt{2} \angle45^\circ \]

\[ V_2 = 10\sqrt{2} \angle45^\circ = V \left[ \frac{5}{5+2} \right] \]

\[ \therefore V = 2V_2 \Rightarrow V = 20\sqrt{2} \angle45^\circ \]

5) For what clock frequency, the n/w acts as ideal current source block A & B.

Ideal current source \( \Rightarrow Z_{eq} = 0 \)

\[ Z_{eq} = \left( \frac{j\omega}{16} \right) \frac{1}{1\left( \frac{-j}{\omega} \right)} \]

\[ = \frac{j\omega}{16\left( \frac{\omega}{16} - \frac{1}{\omega} \right)} \rightarrow 0 \]

\[ \therefore \frac{\omega}{16} = \frac{1}{\omega} \Rightarrow \omega^2 = 16 \Rightarrow \omega = 4 \]
1) \[ V_{TH} = (100 \angle 0^\circ) \cdot \frac{j4}{j5 + j4} = \frac{400}{7} \angle 0^\circ \ V \]

2) \[ Nodal: \quad \frac{V - (30 \angle 45^\circ)}{1-j} + \frac{V - (30 \angle -45^\circ)}{1+j} + \frac{V}{1} = 0 \]

\[ \implies \frac{[V - (30 \angle 45^\circ)](1+j) + [V - (30 \angle -45^\circ)](1-j) + V}{2} = 0 \]

\[ 4V = (30 \angle 45^\circ)(1+j) + (30 \angle -45^\circ)(1-j) \]

\[ = (30 \sqrt{2} \angle 90^\circ) + (30 \sqrt{2} \angle -90^\circ) \]

\[ 4V = 0 \implies V = 0 \implies i = 0 \ A \]

3) What is the max. power transferred to the load impedance.

It is a general case prob.

\[ Z_{TH} = (5 + j5) / (5 - j5) \]

\[ = \frac{5 - j5}{5} \ \Omega \]

For \( P_{max} \):

\[ Z_L = Z_T^* = 5 + j5 \ \Omega \]

\[ P_{max} = \frac{1}{4l} \frac{V_{TH}^2}{R_{TH}} \]
\[ V_m = (50 \text{ Volts}) \cdot \left[ \frac{5 + j5}{\sqrt{50}} \right] = 50 \sqrt{2} \angle 45^\circ \text{ V} \]

\[ \text{P}_{\text{max}} = \frac{(50 \text{ Volts})^2}{4 \times 5} = 250 \text{ W} \]
7) \[ \text{Find } I_2 \]

\[
\begin{align*}
I_1 + jI_2 &= 0 \\
-I_1 + I_2 &= 1 \\
(1 + j)I_2 &= 1 \\
I_2 &= \left(\frac{1}{1+j}\right) \text{ A}
\end{align*}
\]

8) \[ V_1 = 220 \text{ V}, \ V_2 = 122 \text{ V}, \ V_3 = 136 \text{ V} \]

\( \text{(a) Find Load Power Factor} \)

\( \text{(b) If } R_L \text{ is } 5 \Omega, \text{ what is the average power in the load (i.e. active power in watts)?} \)

Load Power Factor: \( \cos \phi_L = \cos (\theta_2, I) \)

N/W Power Factor: \( \cos \phi = \cos (\theta_1, I) \)

\( \rightarrow \) This is based on phasor diagram:

\[ V_1(220) \rightarrow V_2(122) \rightarrow V_3(136) \]

Formula: \[ R_L = \sqrt{R_1^2 + R_2^2 + 2R_1R_2 \cos(\theta_1, \theta_2)} \]

\[ 220 = \sqrt{(122)^2 + (136)^2 + 2(122)(136) \cos \phi_L} \]

\[ \cos \phi_L = 0.45 \text{ (lag)} \]
Now, \[ R_L = 58 \]

\[ P = \frac{1}{2} |V_x|^2 = (|I|)^2 R = (|I|)^2 |V| \]

Also, \[ \cos \phi_L = \frac{V_x}{V} \]

\[ V_x = V_3 \cos \phi_L = 136 \times 0.8 = 108.8 \]

\[ P = \frac{(61.2)^2}{5} = 750 \text{ W} \]

9) \[ I_{11} = I_{23} = 10 \text{ A} \]

determine \[ IR \& R \]

\[ IR = \sqrt{160 - 64} = 536 \]

\[ I_R = 6 \text{ A} \]

From phasor diagram:

\[ I_2 = 8 + 8 = 16 \text{ A} \]

10) Verify Reciprocity theorem in final voltage \[ V \]

**Step 1**

\[ V = \left[ 5 L 45^\circ \left( \frac{2 + j2}{3 + j1} \right) \right] (-i1) \]

**Step 2**

Reciprocal voltage

\[ V = (2 + j2) \left[ \frac{5 L 45^\circ (j1)}{(3 + j1)} \right] \]
So we find complex power $S^*$ in each element.

<table>
<thead>
<tr>
<th>Element</th>
<th>$V$</th>
<th>$I$</th>
<th>$I^*$</th>
<th>$VI^*$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_s$</td>
<td>10L0°</td>
<td>2.23L-145.4°</td>
<td>22.3L45.4°</td>
<td>Source -&gt; Apparent Power</td>
<td></td>
</tr>
<tr>
<td>$I_s$</td>
<td>10L-3.63</td>
<td>5L30°</td>
<td>5L30°</td>
<td>50L-33.63°</td>
<td>Source -&gt; Apparent Power</td>
</tr>
<tr>
<td>$R$</td>
<td>8.34L26.31</td>
<td>2.18L26.31</td>
<td>23.18L0°</td>
<td>Sink -&gt; Active Power</td>
<td></td>
</tr>
<tr>
<td>$L$</td>
<td>4.46L-55.4°</td>
<td>2.23L-145.4°</td>
<td>9.94L90°</td>
<td>Sink -&gt; Reactive Power (+80°)</td>
<td></td>
</tr>
<tr>
<td>$C$</td>
<td>5L-60°</td>
<td>5L30°</td>
<td>5L30°</td>
<td>25L-120°</td>
<td>Sink -&gt; Reactive Power (-90°)</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
KVL: & \quad -[10\angle0°] + j2I_1 + 3I_1 + 3(5L30°) = 0 \\
\therefore & \quad (3+j2)I_1 + 10\angle0° + 2.99 + 7.5j = 0 \\
& \quad I_1 = \frac{-10\angle0° + 5L30° + 2.99 + 7.5j}{3 + j2} = 2.23L-145.4° \\
\end{align*}
\]

\[
\begin{align*}
I_R & = I_1 + 5L30° = 2.78L26.31 \\
V_R & = 3(I_R) = 8.34L26.31 \\
I_L & = 2.23L-145.4° \\
V_L & = (52)I_L = (2L90°)(2.23L-145.4°) \\
& = 4.46L-55.4° \\
V_C & = (52)I_C = (1L-90°)(5L30°) \\
& = 5L-60°
\end{align*}
\]
To verify Tellegen Theorem,
\[ \sum \vec{V} \vec{I}^* \text{Source} = \sum \vec{V} \vec{I}^* \text{Sink} \]

DVS = \[ \begin{bmatrix} 22.3 \angle 45.4^\circ \\ 150 \angle -33.6^\circ \end{bmatrix} \]

RVS = \[ \begin{bmatrix} 23.1 \angle 20^\circ \\ 0.94 \angle 90^\circ \\ 25 \angle -90^\circ \end{bmatrix} \]

= 27.7 \angle -32.8^\circ \, \text{VA's}

12) Find the shift in the neutral voltage using Millman's theorem for the unbalanced 3-phase circuit shown.

\[ V' = \frac{\left( \frac{100 \angle 0^\circ}{5} \right) \left( \frac{100 \angle -120^\circ}{5 + j 5} \right) \left( \frac{100 \angle -240^\circ}{2 - j 2} \right)}{\frac{1}{5} + \frac{1}{5 + j 5} + \frac{1}{2 - j 2}} \]

\[ V' = V_{No} \text{ shift in neutral voltage} \]
\[ V = \frac{20\angle 0^\circ + 14.14\angle -165^\circ + 35.36\angle -195^\circ}{0.2 + 0.1 - 0.1i + 0.25 + 0.25i} \]

\[ = \frac{-27.81\angle 168.82^\circ}{4.972\angle 153.7^\circ} \]

\[ = 5.6972\angle 153.7^\circ \, V \]

12) Find change in current in capacitor when 1 ohm resistor is increased to 3 ohms.

\[ I_{1.5r} = \frac{5060}{1 + [(2 + j2)/(1 + j5)]} \]

\[ = \frac{5060}{1 + 0.4 - 1.2} \]

\[ = 27.11\angle 46.6^\circ \]

Now, find compensated voltage.

\[ V_c = I [A2] \]

\[ = [27.11\angle 40.6^\circ][3 - i] \]

\[ = 54.22\angle 40.6^\circ \]

Compensated flow.

\[ I = \frac{54.22\angle 40.6^\circ}{3 + [(2 + j2)/(1 - j1)]} \]

\[ = 15.80\angle 60^\circ \]

\[ I_{C_{\text{cessed}}} = \frac{\frac{2 + j2}{2 + j2 - j1}}{2 + j2} \]

\[ = 18.96\angle 78^\circ \, A \]
**Duals & Duality**

Two cells are duals of each other if the mesh eqn that characterize one of them has the same mathematical form as the nodal eqn that characterize the other.

**Principle of Duality:**

Identical behaviour & pattern observed for 2. voltages & currents of 2 independent units illustrate the principle of duality.

**Eqn**

![Voltage equations](image)

**Mesh - KVL**

\[-V_s + V_R + V_L + V_C = 0\]

\[V_s = iR + L \frac{di}{dt} + \frac{1}{C} \int i dt\]

**Nodal - KCL**

\[-I_s + I_G + I_C + I_L = 0\]

\[I_s = V_G + C \frac{dV}{dt} + \frac{1}{L} \int i dt\]
Some Dual Elements

\[ V \leftrightarrow I \]
\[ V(t) \leftrightarrow i(t) \]
\[ V_{\text{sin}} \leftrightarrow I_{\text{sin}} \]
\[ R \leftrightarrow G \]
\[ L \leftrightarrow C \]
\[ \text{KVL} \leftrightarrow \text{KCL} \]
\[ \text{Series} \leftrightarrow \text{Parallel} \]
\[ \text{Mesh} \leftrightarrow \text{Node} \]
\[ \& \]
\[ T \leftrightarrow T \]
\[ \text{Tree} \leftrightarrow \text{Co-Tree} \]
\[ * \text{Twig} \leftrightarrow \text{Link/Chord} \]

\[ \frac{\text{di}}{\text{dt}} \leftrightarrow \frac{\text{dv}}{\text{dt}} \]
\[ \text{Sud} \leftrightarrow \text{Sid} \]
\[ \text{O.C} \leftrightarrow \text{S.C} \]
\[ \text{Thevenin} \leftrightarrow \text{Norton} \]
\[ * \phi \leftrightarrow q \]
\[ Y \leftrightarrow \Delta \]

\[ \text{Cutset} \leftrightarrow \text{Tie-set} \]
\[ Z \leftrightarrow Y \]
\[ X \leftrightarrow B \]

\[ \text{s/w in series} \leftrightarrow \text{s/w in parallel} \]
\[ \text{(getting closed)} \leftrightarrow \text{(getting opened)} \]

\[ \text{polarity (voltage)} \leftrightarrow \text{direction (current)} \]

\[ \text{Dual of 2}\ \text{Ohm resistance} \Rightarrow \text{2 Seimens} \]
\[ \text{Conductance} \]
Construct the dual of the circuit shown below and verify by writing new equations.

Mesh
\[-V + i_1 R + L \left( \frac{di_1}{dt} - \frac{di_2}{dt} \right) = 0\]
\[i_2 = -I\]

Nodal
\[-I + G V + C \left( \frac{dV_1}{dt} - \frac{dV_2}{dt} \right) = 0\]
\[V_2 = -V\]

Mesh
\[i_1 = 5\]
\[\frac{1}{3} \left( i_1 - i_2 \right) + \left( i_3 - i_1 \right) = 0\]
\[2 \left( i_1 - i_2 \right) + 2 \left( \frac{dv_1}{dt} - \frac{dv_2}{dt} \right) + \frac{1}{7} \int i_4 dt + 10 + 8 V_3 = 0\]
\[i_3 - i_4 = aV_x\]
\[V_3 = 2 \left( \frac{dv_1}{dt} - \frac{dv_4}{dt} \right)\]

Nodal
\[V_1 = s\]
\[\frac{1}{3} \left( V_2 - V_1 \right) dt + 4 \left( V_2 - V_3 \right) = 0\]
\[i_1 \left( V_3 - V_2 \right) + 2 \left( \frac{dv_1}{dt} - \frac{dv_4}{dt} \right) + 1 \left( V_4 - V_1 \right) + 6 V_4 + 6 \frac{dV_3}{dt} + \frac{1}{7} \int V_4 dt + 10 + 8 V_3 = 0\]
\[i_x = 2 \left( \frac{dv_1}{dt} - \frac{dv_4}{dt} \right)\]
NETWORK TOPOLOGY or GRAPH THEORY

Topology

Topology is a branch of geometry applicable to electrical clsbs where even by bending, stretching, swapping, thrashing, tying in knots, making clsb upside down, etc will not disturb the clsb property.

\[ \text{Topology} \]

\[ \begin{align*}
&\quad\quad 1.8 \quad 2\Omega \\
&\quad\quad \frac{1}{3}\Omega \quad \frac{1}{1}\Omega \\
&\quad\quad 0\Omega \quad 0.5\Omega \\
&\quad\quad 10V \\
\end{align*} \]

\[ \text{Conversion} \]

\[ \text{1F} \quad 0\Omega \quad 2\Omega \quad 0\Omega \quad 10V \]

\[ \Rightarrow \text{Shape of a clsb ideally will not affect circuit analysis.} \]

Graph

A graph is a skeleton representation of a clsb where every element is suppressed by its nature & represented as a simple line-segment.

NOTE

Ideal voltage source \( \rightarrow \) S.C

ideal current \( \rightarrow \) O.C
To simplify the order of matrix we generally consider principal node only.

- **Nodes** → vertices
  - Branches → edges

- **Nodes are numbers** → 0, 1, 2, 3, ...
  - Branches are named → a, b, c, ...

Construct the correct graph of the nil shown below:

![Graph Diagram]

**Node (n)**
- \( n = 4 \)

**Branch (b)**
- \( b = 6 \)

**Mesh (m) → independent loops**
- \( m = 3 \rightarrow [6 - (4 - 1)] \)
- \( l = 3 + 4 = 7 \)

**Loop (L)**

**Path**

A path is a traversal from one node to another node without crossing the same node twice.

**No. of possible path for the above graph from node 0 to node 3 is 5**

1 → 2 → 0 → 3, 2 → 3, 0 → 3, 2 → 3, 0 → 3
Sub graph

A sub-graph consists of some nodes & branches of the main graph.

Even a single edge can be sub-graph of the main graph.

Directed Graph

A graph is said to be directed if every edge is given a reference dir which is indicated by placing an arrow on every branch.

Note:

This reference orientation need not necessarily indicate current dir.

Connected Graph

A graph is said to be connected if there exist atleast one path from every node to every other node.
Complete Graph / Completely Connected Graph.
A graph is said to be a complete graph \( G \) if there exists a direct path from every node to every other node.

Unconnected Graph :-
1) Wireless communications nets
2) Magnetic circuits

The minimum no. of edges to make a graph complete with \( n \) nodes is \( \binom{n}{2} \).

<table>
<thead>
<tr>
<th>Nodes</th>
<th>Graph</th>
<th>Edges</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td><img src="image1" alt="Diagram" /></td>
<td>( \frac{2(2-1)}{2} = 1 )</td>
</tr>
<tr>
<td>3</td>
<td><img src="image2" alt="Diagram" /></td>
<td>( \frac{3(3-1)}{2} = 3 )</td>
</tr>
<tr>
<td>4</td>
<td><img src="image3" alt="Diagram" /></td>
<td>( \frac{4(4-1)}{2} = 6 )</td>
</tr>
<tr>
<td>5</td>
<td><img src="image4" alt="Diagram" /></td>
<td>( \frac{5(5-1)}{2} = 10 )</td>
</tr>
</tbody>
</table>
Tree

A Tree is a subgraph of a main graph which connects all the nodes without forming closed loops.

The rank of a tree with 'n' nodes is (n-1).

Any corresponding tree of a given graph with 'n' nodes will have (n-1) edges.

The number of trees = \[
\begin{cases} 
{n-2 \choose n} & \text{for } n > 2 \rightarrow \text{complete graph only} \\
\det([A_r][A_r]^T) & \text{for any graph}
\end{cases}
\]

where, \([A_r] = \text{Reduced Incidence Matrix}\)

The branch of a tree is specifically called as a Twig indicated by thick line segment.

Any tree with 'n' nodes has (n-1) twigs.

Co-Tree (Complement of Tree)

The set of all branches other than Tree branches in a graph collectively form a co-tree.

Link / Chord:

The branch of Co-Tree is specifically called as a link which is indicated by dotted line. For any corresponding Co-Tree we have b-(n-1) links.
\[ \text{GRAPH} = \underbrace{\text{TREE} + \text{CO-TREE}}_{\text{edges}} = b - (n-1) = (n+1) \]

1) The no. of possible f-trees for the new graph shown below is and draw separately.

\[ \rightarrow \text{This is a complete graph} \]

\[ \rightarrow \text{No. of Trees} = n^{(n-2)} = 4^2 = 16 \]
Incidence Matrix $[A]$

It is the matrix that gives relation blur no. of modes & no. of branches & the orientation of a particular branch w.r.t a node.

The order of the matrix is $(n \times b)$ or $(V \times E)$

The rank of Incidence Matrix with $n$ nodes is $(n-1)$

The elements of this matrix $[A] = [a_{ij}]_{n\times b}$

- $a_{ij} = +1$ if $j$th branch is incident with $i$th node
- $a_{ij} = -1$ if incidented towards
- $a_{ij} = 0$ if not incident

Construct the complete Incidence Matrix for the oriented graph shown below:

![Graph Diagram]

$[A] = \begin{bmatrix}
-a & b & c & d & e & f \\
-1 & +1 & +1 & 0 & 0 & 0 \\
0 & -1 & -1 & 1 & 0 & 0 \\
0 & 0 & -1 & +1 & 1 & 0 \\
+1 & 0 & 0 & 0 & -1 & 1 \\
\end{bmatrix}_{5\times 6}$

The algebraic sum of the elements of every column vertically is zero
1) The determinant of Incidence Matrix of a closed loop graph is 0

Incidence matrix of a closed loop graph is of order \( n \times n \)

\[
[A] = \begin{bmatrix}
-1 & 0 & 1 \\
0 & -1 & -1 \\
1 & 1 & 0
\end{bmatrix}
\]

\[
\text{det} [A] = -1((-1)) + (1) = -1 + 1 = 0
\]

In the Incidence Matrix of two independent n/w graphs are identical then they are said to obey the principle of Isomorphism.

Reduced Incidence Matrix \([Ar]\)

If one of the node in a given graph is considered as reference & that particular row is neglected while writing the incidence matrix, then it is a reduced incidence matrix. Its order is \((n-1) \times b\)

In computer methods of electrical n/w analysis, by considering \(Ar\) the memory space requirement & iteration time for solutions will be decreased.

From above graph if node 4 is considered as ref. & that particular row \(4\) is neglected, then the reduced incidence
The matrix is:

\[
[A_r] = \begin{bmatrix}
-1 & 1 & 0 & 0 & 0 \\
0 & -1 & 0 & -1 & 1 \\
0 & 0 & -1 & 1 & 0
\end{bmatrix}
\]

2) Construct the oriented graph of a new where Incidence Matrix is given as:

\[
\begin{bmatrix}
-1 & 0 & 0 & 0 & 1 & 0 \\
0 & -1 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 0
\end{bmatrix}
\]

→ The given is an \([A_r]\)

So make it (complete \([A]\))

\[
\begin{bmatrix}
1 & -1 & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 1 & 0 \\
-1 & 0 & 0 & 1 & 0 & 0
\end{bmatrix}
\]

→ The new graph has nodes = 4, edges = 6

3) The no. of possible trees from new graph shown below is ___ & draw them separately.

→ Just connected graph.

No. of trees = \(|\det([A_r][A_r]^T)|
\]
\[
\begin{bmatrix}
1 & 0 & 0 & -1 & 0 \\
0 & -1 & 0 & 0 & -1 \\
0 & 0 & -1 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & -1 \\
-1 & 0 & 1 \\
0 & -1 & 1
\end{bmatrix}
\]

\[
[A_r][A_r]^T = \begin{bmatrix}
2 & 0 & -1 \\
0 & 2 & -1 \\
-1 & -1 & 3
\end{bmatrix}
\]

\[
\det [A_r][A_r]^T = 2(6-1) - 1(2) = 10 - 2 = 8 \text{ possible trees.}
\]
Concept of Fundamental Loops and Tie Set Currents:

Fundamental loops are closed paths of the graph which are formed by only one link plus rest of them as twigs.

The no. of F-loops for any given graph = no. of links i.e. \( b - (n-1) \)

These fundamental loops currents are called Tie-set current & their orientation is governed by the link in it.

\[ \text{Eq:} \]

\[ \begin{align*}
\text{No. of F-loops} & = 6 - (4-1) = 3 \\
FL_1 & = a, b, e \rightarrow i_1 \) \\
FL_2 & = b, c, d \rightarrow i_2 \)
\]

\[ \text{FL}_3 = \text{def} \rightarrow i_3 \]

\[ \begin{align*}
\text{FL}_1 & = a, b, e \rightarrow i_1 \) \\
\text{FL}_2 & = a, c, f \rightarrow i_2 \)
\]

\[ \text{FL}_3 = \text{def} \rightarrow i_3 \]
Tie-Set Matrix \([M]\):

It is a matrix that gives the relation between branch currents and tie-set currents where every branch current can be expressed in terms of tie-set currents.

The order of the matrix is \((\text{lines} \times \text{branches})\)

\[ [b-(n-1)] \times b \]

\(n\) is the number of tie-set matrices possible for any graph = no. of trees.

The elements of the matrix \([M] = [a_{ij}]\) where

\(a_{ij} = +1\) if \(j\)th branch current is incident with \(i\)th tie-set current and oriented in same dir.

\(a_{ij} = -1\) if incident & opp.

\(a_{ij} = 0\) if not incident.

\(\Delta\) Construct the Tie-Set Matrix & the write the equilibrium eqn in std. KVL form for new graph shown below by considering \(a, b, c\) as tree branches.

\[ \begin{align*}
\text{No. of f. loops} &= 5 - (4 - 1) \\
&= 2 \\
f_1 \rightarrow a, c, d \rightarrow i_1 \\
f_2 \rightarrow a, b, e, e \rightarrow i_2
\end{align*} \]
Equilibrium Equations

Set $i_a, i_b, i_c, i_d, i_e \rightarrow$ Branch Currents
$V_a, V_b, V_c, V_d, V_e \rightarrow$ Branch voltages

Set I KVL \[ \rightarrow \]

\[
[M][V_b] = [0]
\]

\[
\begin{bmatrix}
1 & 0 & -1 & 0 & 0 \\
0 & 1 & 1 & -1 & 0 \\
1 & -1 & 0 & 1 & 0 \\
0 & 1 & -1 & 0 & 1
\end{bmatrix}_{(2 \times 5)}
\begin{bmatrix}
V_a \\
V_b \\
V_c \\
V_d \\
V_e
\end{bmatrix}_{5 \times 1} =
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}_{5 \times 1}
\]

Set II Relation b/w $j$ & $i$ \[ \rightarrow \]

\[
[M]^T[I_i] = [J_b]
\]

\[
\begin{bmatrix}
1 & 1 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 1 \\
1 & 0 & 1 & 1 & 1 \\
1 & 1 & 0 & 1 & 1 \\
1 & 1 & 1 & 0 & 1
\end{bmatrix}_{5 \times 2}
\begin{bmatrix}
i_1 \\
i_2
\end{bmatrix}_{2 \times 1} =
\begin{bmatrix}
j_a \\
j_b \\
j_c \\
j_d \\
j_e
\end{bmatrix}_{5 \times 1}
\]

\[
\begin{bmatrix}
i_1 + i_2 \\
i_2 \\
-i_1 - i_2 \\
i_1 \\
i_2
\end{bmatrix} =
\begin{bmatrix}
j_a \\
j_b \\
j_c \\
j_d \\
j_e
\end{bmatrix}_{5 \times 1}
\]
In graph theory, if a graph is given, we consider every edge as a local set of reference (by default) \( j_b + v_b \).

**Concept of cut-set:**

A cut set represents a set of branches which when removed in a graph can be divided into 2 parts.

*Note* The no. of cut-sets simply represent the no. of possible ways that a graph can be divided into 2 parts.

1. Which of the following sets of branches represent a proper cut set for the graph shown below?

   - (a) \( \{a, c, d\} \)
   - (b) \( \{b, d\} \)
   - (c) \( \{a, c, e\} \)
   - (d) \( \{c, d, e\} \)
Concept of fundamental cut-set & cut-set voltages:

A fundamental cut-set is a cut through of a graph which can divide it into 2 parts in such a way that if any of the cut in path of cutting it should cut only one twig & rest of them as links.

The no. of f-cut-set = no. of twigs (i.e. n-1)

These f-cut sets form iso potential lines & their voltages are termed as cut set voltages.

The orientation of f-cut sets are governed by the twig in it.

**GRAPH**

**TREE-1**

No. of f-cut set = 4-1 = 3

\[ \text{fC}_1 \rightarrow a, b, c \rightarrow e_1 \]
\[ \text{fC}_2 \rightarrow c, d, f \rightarrow e_2 \]
\[ \text{fC}_3 \rightarrow a, e, f \rightarrow e_3 \]
Cut-set Matrix $[C]$:

It is a matrix that gives relation b/w branches, & cut-set vltgs, where every branch vltg can be expressed in terms of cut-set vltgs.

The order of this matrix $= (n-b) \times b$ i.e. $(n-b) \times b$

The no. of possible cut-set matrix for any given graph $= \text{no. of trees}$.

The elements of this matrix $[C] = [a_{ij}]_{(n-b)\times b}$

where $a_{ij} = 1$ if $j^{th}$ branch vltg is incident to $i^{th}$ cut set vltg & oriented in same dir.

$a_{ij} = -1$ if incident & opp.
\( a_{ij} = 0 \quad \text{if not incident.} \)

1) Construct the cut-set matrix \( \mathbf{C} \) and write the equilibrium equations for the oriented graph shown below by considering branches \( a, b, c \) as tree branches.

\[
\begin{align*}
\text{No. of cut sets} & = 4 - 1 = 3 \\
\mathbf{f}_1 &= a, d, e \rightarrow e' \\
\mathbf{f}_2 &= b, e \rightarrow e' \\
\mathbf{f}_3 &= c, d, e \rightarrow e' \\
\end{align*}
\]

\[
\mathbf{C} = 
\begin{bmatrix}
1 & 0 & 0 & -1 & -1 \\
0 & 1 & 0 & 0 & -1 \\
0 & 0 & 1 & 1 & 1
\end{bmatrix}
\]

Let,
\( j_a, j_b, j_c, j_d, j_e \rightarrow \) Branch currents
\( V_a, V_b, V_c, V_d, V_e \rightarrow \) Branch voltages.

\[
\begin{align*}
\text{KCL} \quad & \mathbf{C} \mathbf{j} = 0 \\
\begin{bmatrix}
1 & 0 & 0 & -1 & -1 \\
0 & 1 & 0 & 0 & -1 \\
0 & 0 & 1 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
j_a \\
j_b \\
j_c \\
j_d \\
j_e
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\end{align*}
\]
\[
\begin{bmatrix}
3a - 3d - 3e \\
\delta_b - 3e \\
\delta_c + \delta_d + 3e
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\]

S-2

Relation btw \( V' \) & \( \delta' \) \[ \downarrow \]

\[
[c]^T [e_t] = [V_b]
\]

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
-1 & 0 & 1 \\
-1 & -1 & 1
\end{bmatrix}_{5 \times 3}
\begin{bmatrix}
[e_1] \\
[e_2] \\
[e_3]
\end{bmatrix}_{3 \times 1}
= \begin{bmatrix}
V_a \\
V_b \\
V_c \\
V_d \\
V_e
\end{bmatrix}_{5 \times 1}
\]

\[
\begin{bmatrix}
e_1 \\
e_2 \\
e_3 \\
-e_1 + e_3 \\
-e_1 - e_2 + e_3
\end{bmatrix}
= \begin{bmatrix}
V_a \\
V_b \\
V_c \\
V_d \\
V_e
\end{bmatrix}
\]

2) Mention the relation btw Tie-set & cut-set matrices.

\( \Rightarrow \) For the same given \( n/W \); for that particular tree, for that particular orientation only we can compare Tie-set & Cut-set matrices.
Tie set → Links → KVL → Mesh (f-loops)

\[
[M] = e_1 \begin{bmatrix} a & b & c \\ 1 & 0 & -1 \end{bmatrix} e_2 \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 \end{bmatrix} \]

\[
\downarrow \quad \downarrow \\
M_{\text{twigs}} \quad U_{\text{twigs}}
\]

Cut-set → Twigs → KCL → Nodeal

\[
[C] = e_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} e_2 \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} e_3 \begin{bmatrix} -1 & -1 \\ 0 & 1 \end{bmatrix} \]

\[
\downarrow \quad \downarrow \\
U_{\text{twigs}} \quad C_{\text{links}}
\]

Also,

\[
[M_{\text{twigs}}] = -[C_{\text{links}}]^T
\]

\[
[C_{\text{links}}] = -[M_{\text{twigs}}]^T
\]
Application of New Topology in Electrical circuit Analysis:

Concept of sta. branch:

\[ V_b = Z_b [j_b + i_s] - v_s \]

But, 
\[ [M][V_b] = 0 \]
\[ \therefore MZ_b j_b + MZ_b i_s - MV_s = 0 \]

But, 
\[ [M]^T[I_e] = [T_b] \]

\[ [M][Z_b][M]^T[I_e] = [M][V_s] - [M][Z_b][i_s] \]

Solve this

Then branch currents can be calculated

\[ [M]^T[I_e] = [T_b] \]

\[ \text{Substitute} \]

\[ \text{Thus} \]

\[ \rightarrow \text{Final ans.} \]
[II] Solution by KCL Eq\(^n\) [Cut-set]

\[ I_b = Y_b (V_b + V_s) - I_s \]

Admittance form.

But,

\[ [C] [I_b] = 0 \]

\[ C Y_b V_b + C Y_b V_s - C I_s = 0 \]

But,

\[ [C]^T [e_+] = [V_b] \]

\[ [C]^T [Y_b] [C]^T [e_+] + [C] [I_b] - [C] [Y_b] [V_s] \]

Solve this.

Then branch voltages can be determined by:

\[ [C]^T [e_+] = [V_b] \]

Substitute this: Final answer.

1) Solve the nLw to find branch currents by writing the eq\(^n\) in std KVL form.

2) Solve the nLw to find all the branch voltages by writing eq\(^n\) in std KCL form.
1) TIE SET

\[
[M][Z_b][H]^{-1} = [M][V_s] = [M][Z_b][I_s]
\]

\[
[M] = \begin{bmatrix}
        a & b & c & d & e & f \\
        0 & 0 & 1 & 0 & 0 & 1 \\
        0 & 1 & 0 & -1 & 0 & -1 \\
        0 & 0 & 1 & 0 & -1 \\
      \end{bmatrix}_{3 \times 6}
\]

\[
[Z_b] = \begin{bmatrix}
        1 & 0 & 0 & 0 & 0 & 0 \\
        0 & 0 & 1 & 0 & 0 & 0 \\
        0 & 0 & 0 & 1 & 0 & 0 \\
        0 & 0 & 0 & 0 & 1 & 0 \\
        0 & 0 & 0 & 0 & 0 & 1 \\
      \end{bmatrix}_{6 \times 6}
\]

\[
[I_s] = \begin{bmatrix}
        i_1 \\
        i_2 \\
        i_3 \\
      \end{bmatrix}_{3 \times 1}
\]

\[
[V_s] = \begin{bmatrix}
        +2 \\
        0 \\
        0 \\
      \end{bmatrix}_{6 \times 1}
\]

\[
[I_s] = \begin{bmatrix}
        0 \\
        0 \\
        0 \\
      \end{bmatrix}_{6 \times 1}
\]

\[
\begin{bmatrix}
  1 & 0 & 0 & 2 & 2 & 0 \\
  0 & 1 & 0 & -2 & 0 & -2 \\
  0 & 0 & 1 & 0 & -2 & 2 \\
\end{bmatrix}_{3 \times 6}
\]

\[
\begin{bmatrix}
  1 & 0 & 0 \\
  0 & 1 & 0 \\
  0 & 0 & 1 \\
\end{bmatrix}_{6 \times 3}
\]

\[
\begin{bmatrix}
  5 & -2 & -2 \\
  -2 & 5 & -2 \\
  -2 & -2 & 5 \\
\end{bmatrix}_{3 \times 3}
\]

\[
\begin{bmatrix}
  i_1 \\
  i_2 \\
  i_3 \\
\end{bmatrix}_{3 \times 3}
\]
RHS \([Y][I_s] = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}\) 

Final \(LHS\)
\[
\begin{bmatrix}
5i_1 - 2i_2 - 2i_3 \\
-2i_1 + 5i_2 - 2i_3 \\
-2i_1 - 2i_2 + 5i_3
\end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}
\]

Solving
\[i_1 = \frac{5}{7} A, \quad i_2 = \frac{4}{7} A, \quad i_3 = \frac{4}{7} A\]

Then, branch currents can be calculated by
\([M]^T[I_c] = [J_b]\)

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -1
\end{bmatrix} \begin{bmatrix}
6/7 \\
4/7 \\
4/7
\end{bmatrix} = \begin{bmatrix} j\alpha \\
j\beta \\
j\gamma \\
j\delta \\
j\epsilon \\
j\zeta
\end{bmatrix}
\]

Final \(RHS\)
\[j\alpha = \frac{6}{7}, \quad j\beta = \frac{4}{7}, \quad j\gamma = \frac{4}{7}, \quad j\delta = \frac{2}{7}, \quad j\epsilon = \frac{2}{7}, \quad j\zeta = 0\]

2) \textbf{CUT-SET}

\([C][Y_b][C]^T[I_c] = [C][I_c] - [C][Y_b][E_s]\)
\[ \begin{bmatrix} a & b & c & d & e & f \\ 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{3 \times 6} \]

\[ \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_{3} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/2 \end{bmatrix}_{3 \times 6} \]

\[ \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}_{3 \times 1} \]

\[ \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}_{3 \times 1} \]

\[ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}_{3 \times 1} \]

\[ \begin{bmatrix} -1 & 0 & 1/2 & 0 & 0 \\ -1 & 0 & 0 & 1/2 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{bmatrix}_{3 \times 6} \]

\[ \begin{bmatrix} 1 & -1/2 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}_{3 \times 6} \]

\[ \begin{bmatrix} 5/2 & 1 & 1 \\ 1 & 5/2 & -1 \\ 1 & -1 & 5/2 \end{bmatrix}_{3 \times 3} \]

\[ \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}_{3 \times 1} \]

\[ \begin{bmatrix} -2 & -2 \\ 2 & 2 \\ 0 & 0 \end{bmatrix}_{3 \times 3} \]

\[ \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}_{3 \times 1} \]

Solving

\[ \begin{bmatrix} e_1 = 4/7 \\ e_2 = 4/7 \\ e_3 = 0 \end{bmatrix}_{3 \times 1} \]
Thus, branch voltage can be calculated by

\[
\begin{bmatrix}
-1 & -1 & 0 \\
1 & 0 & -1 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
4/7 \\
4/7 \\
4/7 \\
0 \\
\end{bmatrix}
= 
\begin{bmatrix}
V_a \\
V_b \\
V_c \\
V_d \\
V_e \\
V_f \\
\end{bmatrix}
\]

Final Answer:

This indicates presence of source in branch:

\[
V_a = -\frac{8}{7}V, \quad V_b = \frac{4}{7}V, \quad V_c = \frac{4}{7}V, \quad V_d = \frac{4}{7}V, \\
V_e = \frac{4}{7}V, \quad V_f = 0V
\]

1) Which of the fall set of branches is not a tree for the graph shown below.

(a) aghe  (b) bdegh  (c) defg  (d) abfg  (c) defg  

2) No. of tree = 5

\[
\begin{array}{c}
(\circ) \\
(\circ) \\
(\circ) \\
(\circ) \\
(\circ) \\
\end{array}
\]

3) For the oriented graph shown below, \( p \) and \( q \) are number of tree \( P \) and number of cutset \( q \). Then

(a) \( p = 2 \), \( q = 2 \)  (b) \( p = 4 \), \( q = 4 \)  (c) \( p = 4 \), \( q = 6 \)  (d) \( p = 4 \), \( q = 10 \)
Here numbering indicates that there are 4 nodes only & the centre one is not a node.

No. of trees = 4

No. of cut sets = 6

i.e. no. of ways that the graph can be cut into 2 parts.

Proper Tree:

Let same as of Tree + \[[C, V \rightarrow \text{Twigs}]\]

C & V to be linked with twigs.

L & I to be linked with links.

Spanning Graph:

It is a sub-graph which connects all the nodes.

All trees are spanning graphs, but all spanning graphs are not trees.
Ex

Graph

Graph

\[ \text{Spanning graph} \]

\[ \text{but not Tree} \]

\[ \text{also Tree} \]

\[ \text{A nw with } n \text{ nodes & } b \text{ branches.} \]

\[ \text{has } - \text{ no. of node-pair voltages.} \]

\[ \text{Ans} \quad nC_2 = \frac{n(n-1)}{2}. \]

---

**ELECTRICAL RESONANCE**

Resonance is the freq. response of a circuit when the circuit operates at its natural freq. called resonance freq.

Under resonance total supply voltage & supply current are in phase. So, \( \phi = 0^\circ \)

& \( PF = \cos \phi = 1 \) (UPF)

Under resonance the net impedance of the circuit becomes purely resistive & max power will be transferred to the circuit source.
Resonance can occur in any electrical net provided we have 2 similar but opposite matured energy storage components i.e. LSC.

To undergo a good observable resonance for practical applications we need a good quality in these energy storage components which is measured as Quality factor or figure of merit given by:

$$Q - factor = 2\pi \times \left[ \frac{\text{Max. energy stored per cycle}}{\text{Energy dissipated per cycle}} \right]$$

In practical applications $Q \geq 10$

Resonance phenomenon is useful in designing of passive filters, antennas, receivers, Sonars, etc.

<table>
<thead>
<tr>
<th>Element</th>
<th>Q - Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$\omega$</td>
</tr>
<tr>
<td>$\frac{wL}{R}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{wRC}$</td>
<td></td>
</tr>
</tbody>
</table>
\( Q_0 = \frac{1}{R} \sqrt{\frac{L}{C}} \)

\[
\frac{R}{\omega L} \quad \text{and} \quad \frac{1}{\omega RC}
\]

\( Q_0 = R \sqrt{\frac{C}{L}} \)

(1) Series Resonance:

At resonance: \( \omega = \omega_0 \)
\( \bar{V} \) & \( \bar{I} \) in phase
\( \phi = 0^\circ \); \( Z = R \)

But, \( Z = R + j [X_L - X_C] \)

But at resonance, net reactance = 0
\( \therefore X_L - X_C = 0 \Rightarrow X_L = X_C \)

\( \omega L = \frac{1}{\omega_0 C} \Rightarrow \omega_0^2 = \frac{1}{LC} \)

\( \omega_0 = \frac{1}{\sqrt{LC}} \); \( f_0 = \frac{1}{2\pi \sqrt{LC}} \) Hz.

Graph (1)

\( Z \) vs \( \omega \)
Graph 2

\[ Z = R + j \left[ \omega L - \frac{1}{\omega C} \right] \]
\[ X_{net} \]

Graph 3

\[ Y \text{ vs } \omega \]

\[ Y_{max} \]

Graph 4

\[ I \text{ vs } \omega \]

Low 'R'

High 'R'

Phasor Diagram:

(a) \( \omega = \omega_0 \)

\[ Z = R \]

L is purely real.

'I' in phase with 'V'

\[ \phi = 0^\circ \]

\[ \phi = \theta \rightarrow I \]

(b) \( \omega < \omega_0 \)

\[ Z = R - jX_{net} \]

\[ L \rightarrow R - j \text{ clt} \]

'I' leads 'V'

by \( \phi < 90^\circ \)

(Leading PF)

(c) \( \omega > \omega_0 \)

\[ Z = R + jX_{net} \]

\[ L \rightarrow R - j \text{ clt} \]

'I' lags 'V'

by \( \phi > 90^\circ \)

(Clagging PF)

\[ \phi \]

is \( \theta \) with respect to \( I(y) \)
GRAPH B

At resonance $\omega = \omega_0$

$|X_L| = |X_C|

$|V_L| = |V_C|

$Q$ - Factor at Resonance.

$Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}

But $\omega_0 = \frac{1}{\sqrt{LC}}$

→ Under series resonance net impedance is min, so current is max.
Hence it is called as acceptor clot.

→ At series resonance freq. it is as if the total supply voltage appears across the resistor. Hence series resonance is called voltage amplification clot.
Variation of voltages across passive element with change in freq.

\[ f_C = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}} = \frac{1}{2\pi \sqrt{LC}} \sqrt{1 - \frac{R^2C}{2L}} \]

\[ f_C = \frac{f_0}{\sqrt{1 - \frac{R^2C}{2L}}} \text{ Hz} \]

The freq. at which max. vltg. appears across inductor:

\[ f_L = \frac{1}{2\pi \sqrt{LC - \frac{R^2C^2}{2}}} = \frac{1}{2\pi \sqrt{LC} \left[ 1 - \frac{R^2C}{2L} \right]} \]

\[ f_L = \frac{f_0}{\sqrt{1 - \frac{R^2C}{2L}}} \text{ Hz} \]
From circuit

\[ V = \overline{I} \overline{Z} \Rightarrow \overline{I} = \frac{|V|}{|Z|} \]

But

\[ |Z| = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} \]

\[ |I| = \frac{|V|}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} \]

But at \( \omega = \omega_0 \)

\[ |I| = \frac{|V|}{\sqrt{R^2 + \omega^2}} \Rightarrow \text{maximum} \]

So,

\[ |I_0| = \frac{|V|}{R} \]

So, Power transferred is

\[ P_0 = I_0^2 R = \frac{|V|^2}{R} \text{ W} \]

at \( \omega = \omega_0 \)

\[ V_R = I_R R = I_0 R = \frac{|V|}{R} \cdot R = |V| \]

\[ V_L = +j X_L I = j \omega_0 L I_0 = +j \omega_0 L \cdot \frac{|V|}{R} \]

\[ = +j \left[ \frac{\omega_0 L}{R} \right] |V| \Rightarrow V_L = +j \theta_0 |V| \]

\[ V_C = -j X_C I = -j \frac{1}{\omega C} I_0 = -j \frac{|V|}{\omega C} \frac{1}{R} = -j \left[ \frac{\omega C}{\omega_0 R C} \right] |V| \]

\[ \Rightarrow V_C = -j \theta_0 |V| \]

\( \theta_0 \rightarrow \text{very magnified factor} \)
Bandwidth:

Bandwidth represents the range of frequencies for which the power level in the signal is at half of the signal max. power.

→ Half of max. power frequencies:

\[ \frac{P_0}{2} = \frac{I_0^2 R}{2} = \left[ \frac{I_0}{\sqrt{2}} \right]^2 R = (0.707 I_0)^2 R \]

\( \omega_1 \rightarrow \) lower cut-off freq.

\( \omega_2 \rightarrow \) upper cut-off freq.

\[ \frac{1}{V_1} = \frac{I_0}{\sqrt{2} R} \]

\[ R^2 + \left( \omega L - \frac{1}{\omega C} \right)^2 = 2 R^2 \]

\[ \left( \omega L - \frac{1}{\omega C} \right)^2 = R^2 \Rightarrow \omega L - \frac{1}{\omega C} = \pm R \]

\( \omega = \omega_1 \)

\[ \omega L - \frac{1}{\omega C} = -R \quad \text{(1)} \]

\( \omega = \omega_2 \)

\[ \omega_2 L - \frac{1}{\omega_2 C} = R \quad \text{(2)} \]
Bandwidth:

\[ w_2 - w_1 = \frac{R}{4} \text{ rad/sec} \]

\[ f_2 - f_1 = \frac{R}{2\pi L} \text{ Hz} \]

\[ \text{B.W \propto R} \]

\[ \text{B.W is independent of } R \]

\[ f_0 = \text{fo} \]

\[ \Rightarrow \text{Resonance freq. is geometric mean of Bandwidth freq.} \quad w_0 = \sqrt{w_1w_2} \]

\[ f_0 = \sqrt{f_1f_2} \]

also,

\[ w_0 - w_1 = \frac{B.W}{2} \Rightarrow w_1 = w_0 - \frac{R}{2L} \text{ rad/sec} \]

\[ f_1 = f_0 - \frac{R}{4\pi L} \text{ Hz} \]

\[ \therefore w_2 - w_0 = \frac{B.W}{2} \Rightarrow w_2 = w_0 + \frac{R}{2L} \text{ rad/sec} \]

\[ f_2 = f_0 + \frac{R}{2\pi L} \text{ Hz} \]

\[ \Rightarrow \text{Resonance freq. is independent of resistor (R)}. \]

Selectivity := (S)

\[ \text{Selectivity is the ability of a chet/lnw to distinguish or discriminate desired \& undesired freq.} \]

\[ S \propto \frac{1}{\text{B.W}} \]
\[ S = \frac{f_0}{|f_2 - f_1|} = \frac{\omega_0}{|\omega_2 - \omega_1|} \]

\[ S = \frac{1}{2\pi \sqrt{LC}} \frac{1}{R} \frac{1}{2\pi} = \frac{1}{R \sqrt{C}} = Q_0 \]

The value of selectivity is Q factor under Resonance.

(2) Parallel Resonance:

General ckt

\[ Y_T = \frac{1}{R} + \frac{j\omega L}{X_L} + \frac{1}{j\omega C} \]

\[ = \frac{1}{R} + j\frac{\omega L}{X_C} - j\frac{1}{X_L} = \frac{1}{R} + j \left[ \frac{1}{X_C} - \frac{1}{X_L} \right] \]

At resonance (\( \omega = \omega_0 \))
Net susceptance = 0

\[ \frac{1}{X_C} - \frac{1}{X_L} = 0 \Rightarrow \omega_0 L = \frac{1}{\omega_0 C} \]

\[ \omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s} \quad f_0 = \frac{1}{2\pi \sqrt{LC}} \text{ Hz} \]
**Phasor diagrams:**

(a) \( w = w_0 \)

- \( Y = G \)
- \( L \) pure react.
- "I" in phase with "V".
- \( \phi = 0^\circ \)
- PF = 1 (UP).
- \( \phi = 0^\circ \) \( \rightarrow \) \( V \)
- \( \phi \) is negative wrt \( V \) (ref.)

(b) \( w < w_0 \)

- \( Y = G + j B \) net
- \( L \rightarrow R \) parallel
- "I" leads "V" by \( \phi < 90^\circ \)
- Cogging PL
- \( \phi \) is the word \( V \) (ref.)

(c) \( w > w_0 \)

- \( Y = G + j B \) net
- \( L \rightarrow R \) \& \( C \) parallel
- "I" leads "V" by \( \phi > 90^\circ \)
- Leading PF
- \( \phi \) is the word \( V \) (ref.)
At parallel resonance condition, net impedance is more, so current is minimum. Hence it is called rejector circuit.

At 1 st resonance freq. it is as if the total current flows only through res., hence it is called as current amplification circuit.
So, from circuit

\[ |V_1| = \frac{1}{|I_1| |Z_1|} \Rightarrow |V_1| = \frac{|I_1|}{|V_1|} \]

But

\[ |V_1| = \sqrt{\frac{1}{R^2} + \left(\frac{1}{\omega C} - \frac{1}{\omega L}\right)^2} \]

\[ |V_1| = \frac{|I_1|}{\sqrt{\frac{1}{R^2} + 0^2}} \]

So,

\[ |V_1| = \frac{|I_1|}{R} \]

\( \omega = \omega_0 \)

\[ |V_1| = \frac{|I_1|}{\sqrt{\frac{1}{R^2} + 0^2}} \]

So,

\[ |V_1| = |I_1| \]

\( \omega = \omega_0 \)

\[ I_R = \frac{V_R}{R} = \frac{V}{R} = \frac{|I_1| R}{R} \Rightarrow I_R = |I_1| \]

\[ I_L = \frac{V_L}{jXL} = \frac{V}{j\omega L} = -j \left[ \frac{R}{\omega L} \right] \cdot I \]

\[ \Rightarrow I_L = -j \theta_0 |I_1| \]

\[ I_C = \frac{V_C}{-jX_C} = \frac{V}{-j\omega C} = -j \left[ \omega_0 RC \right] \cdot I \]

\[ \Rightarrow I_C = +j \theta_0 |I_1| \]

\( \theta_0 \to \text{current amplification factor.} \)
Parallel resonance phenomenon is used in design of Band Stop Filter.

Practical Parallel Resonance:

\[ Y_T = Y_1 + Y_2 \]
\[ Y_1 = \frac{1}{Z_1} = \frac{1}{R + jX_L} = \frac{R - jX_L}{R^2 + X_L^2} \]
\[ Y_2 = \frac{1}{Z_2} = \frac{1}{-jX_C} = \frac{j}{X_C} \]

\[ Y_T = \frac{R}{(R^2 + X_C^2)} + j \left[ \frac{1}{X_C} - \frac{X_L}{R^2 + X_L^2} \right] \]

At Resonance \( \omega = \omega_0 \)

\[ \text{Net susceptance} = 0 \]
\[ \frac{1}{X_C} = \frac{X_L}{R^2 + X_L^2} \Rightarrow R^2 + X_L^2 = \omega_0 L \times \frac{1}{\omega_0 C} \]

\[ R^2 + X_L^2 = \frac{L}{C} \]

\[ \omega_0^2 L^2 = \frac{1}{C} - R^2 \Rightarrow \omega_0^2 = \frac{1}{LC} - \frac{R^2}{L^2} \]
\[ \omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \]

\[ f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \]

**Ideal Tank Circuit:**

\[ Y_T = Y_L + Y_C \]

\[ Y_C = \frac{1}{jX_C} \quad (R=0) \]

\[ Y_L = \frac{1}{jX_L} = \frac{j}{X_L} \]

\[ Y_T = +j \left[ \frac{1}{X_C} - \frac{1}{X_L} \right] \]

At resonance \( \omega = \omega_0 \)

\[ \Rightarrow \text{Net reactance } = 0 \]

\[ \frac{1}{X_C} = \frac{1}{X_L} \quad \Rightarrow \quad \omega_0 L = \frac{1}{\omega_0 C} \]

\[ \omega_0 = \sqrt{\frac{1}{LC}} \quad \text{real Hz} \quad f_0 = \frac{1}{2\pi \sqrt{LC}} \]

**Determine \( f_0 \):**

\[ f_0 = \frac{1}{2\pi \sqrt{LC}} \left( \frac{R_L^2 - (\frac{1}{LC})}{R_C^2 - (\frac{1}{LC})} \right) \]
Concept of Dynamic Impedance
(Dynamic Resistance)

1. It is the resistivity offered by the circuit to the input at resonant freq.

2. Series R-L-C circuit:
   \[ Z_{\text{dyn}} = R \]

3. Generally parallel R-L-C circuit:
   \[ Z_{\text{dyn}} = R \]

4. Tank circuit:
   \[ Z_{\text{dyn}} = \frac{L}{RC} \]
   Practically \( Z_{\text{dyn}} \gg R \)

5. Ideal tank circuit:
   \[ Z_{\text{dyn}} = \alpha \rightarrow 0°C \]

6. Two practical coils with internal resistance \( R_1, R_2 \) have \( Q \)-factor \( Q_1, Q_2 \) resp. If these coils are connected in series then total \( Q \)-factor is

\[
Q_T = \frac{W(L_1 + L_2)}{R_1 + R_2} = \frac{W L_1}{R_1} + \frac{W L_2}{R_2}
\]
\[ A_T = \frac{Q_1 R_1 + Q_2 R_2}{R_1 + R_2} \]

The network below acts as:

(a) Low pass filter
(b) High pass filter
(c) Band pass filter
(d) Band stop filter

At \( w = 0 \):

\[ V_0 = V_i \left[ \frac{R_2}{R_1 + R_2} \right] \]

\[ H(\omega) = \frac{V_0}{V_i} = \frac{R_2}{R_1 + R_2} \]

At \( w = \omega_0 \):

\[ V_0 = V_i \left[ \frac{R_2}{R_1 + R_2} \right] \]

\[ H(\omega) = \frac{V_0}{V_i} = \frac{R_2}{R_1 + R_2} \]

At \( w = \omega_0 \):

\[ V_0 = 0 \]

\[ H(\omega) = 0 \]

\[ |H(\omega)| = \frac{R_2}{R_1 + R_2} \]
MAGNETIC CIRCUITS

Charges at rest produce only Electro-static field but charges in motion produce both Electric & Magnetic field.

**Amperes Right Hand Thumb Rule:**

- **Permanent Magnet**
  - L → Rare Earth material

- **Afnico's**
  - L → High Retentivity

**Electro-magnet**

- X = N pole
- Y = S pole

- X = S pole
- Y = N pole
Analogies between Electric & Magnetic circuits

Electric Circuit

1. Voltage / EMF is the 'cause'.
   units: volts

2. Current is the 'effect'.
   units: Ampere (A)

3. Ohm's Law:
   \[ R = \frac{V}{I} \]
   Resistance, units: \( \Omega \)

4. \[ R = \frac{\rho d}{\alpha} \] → Electrical material

5. Electric field intensity:
   \[ E = \frac{V}{d} \] units: volt/m

Magnetic Circuit

1. MMF → Magnetic motive force → cause
   \[ \text{MMF} = N \cdot I \]
   units: Ampere-turns (AT)

2. Flux is the 'effect'.
   \( \Phi \), units: Weber (Wb)

3. Ohm's Law:
   \[ \frac{1}{S} = \frac{\text{MMF}}{\Phi} = \frac{NI}{\Phi} \]
   \( S \) → Resistance, units: \( \frac{AT}{Wb} \)

4. \[ S = \frac{l}{\mu a} \] → Magnetic material

5. Magnetic field intensity:
   \[ H = \frac{\text{MMF}}{l} = \frac{NI}{l} \] units: \( \frac{AT}{m} \)
6. Electric Current Density
\[ J = \frac{I}{A} \]
Unit: \( \frac{A}{m^2} \)

7. Electric Flux Density
\[ D = \varepsilon E \]
\[ \varepsilon = \varepsilon_0 \varepsilon_r \]

8. \[ R_s = R_1 + R_2 \]
\[ \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} \]

8. Magnetic Flux Density
\[ B = \frac{\Phi}{A} \]
Unit: \( \frac{Wb}{m^2} \) (Tesla (T))

9. \[ B = \mu H \]
\[ \mu = \text{Molli} \]

9. \[ S'_6 = S'_1 + S'_2 \]
\[ \frac{1}{S'_p} = \frac{1}{S'_1} + \frac{1}{S'_2} \]

KVL
KCL
VDR (Series)
CDR (Coulomb)

K MMF L
K Flux L
MMF D R (Series)
Flux D R (Nel)
Composite Magnetic Circuits:

Series Magnetic Circuit

\[ \Phi = \frac{E_{\text{iron}}}{M_0 \mu_0 \alpha} \]
\[ \Phi_{\text{air}} = \frac{E_{\text{air}}}{M_0 \mu_0 (1) \alpha} \]

\[ \Phi = \Phi_{\text{iron}} + \Phi_{\text{air}} \]

Faraday's Law of Electromagnetic Induction:

1. "There should be change in the flux linkage with the coil in order to induce emf in it."

2. "The magnitude of this induced emf is proportional to rate of change of flux."
Mathematically Faraday's law is given by:

$$ C = -\frac{d\psi}{dt} $$

But, $$ \psi = N\Phi $$ \implies flux linkage (wb/Turns)

$$ e = -N\frac{d\Phi}{dt} $$

- Rate of change of flux (wb/sec)
- No. of turns
- Ive sign due to Lenz's law
- Induced emf in coil (volts)

$$ C = -N\left[\frac{\Phi_{\text{final}} - \Phi_{\text{initial}}}{\Delta t}\right] $$

**Dynamically Induced emf:**

Ex: Motor, generator.

**Statistically Induced emf:**

Ex: Transformer.

Flux is a function of current

$$ \text{MMF} = Ni = \Phi \frac{d}{i} $$

$$ \frac{2}{i} + \frac{\Phi}{i} = \frac{N}{i} \rightarrow \text{const.} $$

$$ \therefore \frac{d}{i} = k $$ \implies $$ \Phi = ki $$

$$ \Phi \propto i $$ \implies Flux, a few of current
Concept of self induced emf &
Self inductance:

\[ \Phi = \frac{\Phi}{i} \times i \]
\[ \frac{d\Phi}{dt} = \frac{\Phi}{i} \frac{di}{dt} \]
\[ e = -N \frac{d\Phi}{dt} \]

\[ e = -\frac{NI}{i} \frac{di}{dt} \]
\[ C = -L \frac{di}{dt} \text{ volts (} V = -e = +L \frac{di}{dt} \text{)} \]

\[ L = \frac{NI}{i} = \frac{\Psi}{i} \text{ (self inductance H)} \]

\[ e = -L \left[ \frac{i_{\text{final}} - i_{\text{initial}}}{\Delta t} \right] \text{ volts.} \]

\[ \Psi = LI \]

Ohm's Law

Also

MMF = \Phi L
NI = \Phi L
\[ N \left[ \frac{N \Phi}{L} \right] = \Phi S' \]

\[ L = \frac{N^2}{S'} \]

\[ L = \frac{\mu N^2}{l/H} \]

\[ L \propto N^2 \]
Concept of Mutual Inductance &
Mutually Induced Emf:

Self inductance is w.r.t. the same coil &
its own turns, current & flux.
However mutual inductance is w.r.t. sep. of
coils (min. two).

\[ \phi_{12} = k \phi_1 \]

\[ 0 \leq k \leq 1 \]

\[ \phi_{12} = \phi_{12} \times \frac{i_1}{i_1} \]

\[ \phi_{12} = \phi_{12} \cdot i_1 \]

\[ \frac{d\phi_{12}}{dt} = \left[ \frac{\phi_{12}}{i_1} \right] \frac{di_1}{dt} \]

\[ C_2 = -N \frac{d\phi_{12}}{dt} \quad \text{(2)} \]

\[ C_2 = -\left[ N \frac{\phi_{12}}{i_1} \right] \frac{di_1}{dt} \Rightarrow \]

\[ E_2 = -\left[ M_{12} \right] \frac{di_1}{dt} \]

\[ M_{12} = \frac{N_2 \phi_{12}}{i_1} = \frac{k \phi_1 N_2}{i_1} \]

Mutual inductance in \( \mu \).

Also

\[ \phi_{21} = k \phi_2 \]

\[ E_1 = -\left[ N \frac{\phi_{21}}{i_2} \right] \frac{di_2}{dt} \]

\[ \text{Mutually induced emf (volts)} \]
\[ e = -\left[ H_{21} \right] \frac{di_2}{dt} \]
\[ H_{21} = \frac{N_1 \Phi_2}{i_2} = \frac{k \Phi_2 N_1}{i_2} \]

* If, dist. b/w coils & permeability of medium, b/w coils is constant, then:

\[ M_{12} = M_{21} = M \]

\[ L_1 = \frac{N_1 \Phi_1}{i_1} \]
\[ L_2 = \frac{N_2 \Phi_2}{i_2} \]

\[ M = \frac{k \Phi_1 N_2}{i_1} = \frac{k \Phi_2 N_1}{i_2} \quad \text{ Mutual inductance b/w coils.} \]

**Relation b/w self & Mutual inductances**

\[ M \times M = \left[ \frac{k \Phi_1 N_2}{i_1} \right] \times \left[ \frac{k \Phi_2 N_1}{i_2} \right] \]

\[ M^2 = k^2 \left[ \frac{N_1 \Phi_1}{i_1} \right] \left[ \frac{N_2 \Phi_2}{i_2} \right] \]

\[ M^2 = k^2 \left[ L_1 \right] \left[ L_2 \right] \]

\[ M = k \sqrt{L_1 L_2} \]

But, \( 0 \leq k \leq 1 \) \quad \Rightarrow \quad M \leq \sqrt{L_1 L_2}

\[ k = \frac{M}{\sqrt{L_1 L_2}} \quad \text{For ideal transf.} \]
\[ L \rightarrow \text{co. eff. of coupling} \quad k = 1 \]
Energy stored in system of 2-coils:

\[ E_T = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M_{12} i_1 i_2 \]

\[ \rightarrow \text{Mutually adding flux} \]

\[ \rightarrow \text{Mutually opposing flux} \]

Mutual inductance is always a positive quantity.

But, mutually induced emf may be the \( -ve \) or \( +ve \) depending on the correct polarity of mutual induced emf is not possible directly & hence we use Dot convention.

1. If current enters the dotted terminal of the 1st coil, then the polarity of mutual voltage is \(+ve\) at the dotted terminal of 2nd coil.

2. If current leaves dotted terminal of 1st coil, then the polarity of mutual voltage is \(-ve\) at the dotted terminal of the 2nd coil.
Determine the correct magnitude & polarity of the mutual voltage w.r.t. the given reference voltage for the syst. of coils shown below.

\[ e_2 = + \frac{d}{dt}i_1 \]

\[ e_2 = + M \frac{d}{dt}i_1 \]

\[ e_1 = + \frac{d}{dt}i_2 \]

\[ e_2 = - M \frac{d}{dt}i_1 \]

\[ e_1 = - M \frac{d}{dt}i_2 \]
**Coils in Series**

Mutually adding

\[ L_{eq} = L_1 + L_2 + 2M \]

Mutually opposing

\[ L_{eq} = L_1 + L_2 - 2M \]

**Coils in Parallel**

Mutually adding

\[ L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} \]

Mutually opposing

\[ L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} \]

\[ L_1 + M_{13} = M_{12} + L_2 - M_{12} \]

\[ = M_{23} + L_3 + M_{31} - M_{32} \]

\[ = (4+6+5) + 2(10) - 2(2) \]

\[ = 15 + 20 - 4 \]

\[ = 31 \text{ mH} \]

\[ V_1 = 4 + 10 = 14 \]

\[ V_2 = 6 - 2 = 4 \]

\[ V_3 = 5 + 10 - 2 = 13 \]
3) \[ \text{Leg} = (6 - 3 + 2) + (5 - 3 + 1) + (4 + 1 + 2) = 5 + 3 + 7 = 15 \, \text{H} \]

4) \[ 2 + 2 + 2 = 6 \, \text{H} \]

\[ \frac{12 - 4}{3 + 4 + 4} = \frac{8}{11} \]

\[ \text{Leg} = 6 + \frac{8}{11} = \frac{74}{11} \, \text{H} \]

5) Write the complete inductance matrix for the systems of coils below.

\[ \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 7 & 3 & 2 \\ -3 & 8 & -4 \\ 2 & -4 & q \end{bmatrix} \begin{bmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \\ \frac{di_3}{dt} \end{bmatrix} \]
\[ V_{BC} = V_5 + V_M(t) \]
\[ = 5 \frac{d}{dt} (6t) + 4 \frac{d}{dt}(6t) \]
\[ = 50 + 2(4) = 74 \text{ volts} \]

\[ V_{CD} = V_M(t) + V_M(t) \]
\[ = -2 \frac{d}{dt}(6t) - 1 \frac{d}{dt}(10t) \]
\[ = -12 - 10 = -22 \text{ volts} \]

\[ V_{BO} = 74 - 22 = 52 \text{ volts} \]

**Rules [Mutual Vltg]**

1. Which Mesh?
2. Which two dots?
3. + or -
4. Resultant current

\[-V + 5 \frac{d}{dt} i_1 + 8 \left[ \frac{d}{dt} i_1 - \frac{d}{dt} i_2 \right] + 9 \frac{d}{dt} i_1 + 1 \frac{d}{dt} i_2 - 3 \frac{d}{dt} i_2 \]
\[ - 4 \frac{d}{dt} i_1 - 4 \left[ \frac{d}{dt} i_1 - \frac{d}{dt} i_2 \right] = 0 \]
\[ 14x - 6y = V \quad \text{(1)} \]

\[
\begin{align*}
M - 2 & \\
8 \left[ \frac{d^2 i_z}{dt^2} - \frac{d^2 i_z}{dt^2} \right] + G \frac{di_z}{dt} + 7 \frac{di_z}{dt} + 4 \frac{di_z}{dt} + 3 \frac{di_z}{dt} + \frac{di_z}{dt} & = 0 \\
-2 \frac{di_z}{dt} + 3 \left[ \frac{d^2 i_z}{dt^2} - \frac{d^2 i_z}{dt^2} \right] - 2 \frac{di_z}{dt} & = 0 \\
-6x + 23y & = 0 \quad \text{(2)}
\end{align*}
\]

From (1) & (2)

\[ x \left[ 14 - \frac{6 \times 6}{23} \right] = V \]

\[ V = \left[ \frac{286}{23} \right] \frac{di_z}{dt} \implies \log = \frac{286}{23} \text{ mH.} \]

8)

Using Right Hand Thumbs rule we decide the dir of flux

\[ L \times y = 9 \]

\[ v_1 = 4 - (1+2) = 5 \]
\[ v_2 = 10 - (1-3) = 6 \]
\[ v_3 = 5 + 2-3 = 4 \]

15 mH

Find \( L \times y \):\[
L_{11} = L_{22} = L_{33} = L_{44} = 5 \text{ mH} \\
M_{12} = M_{23} = M_{34} = 2 \text{ mH} \\
M_{13} = M_{14} = M_{24} = 1 \text{ mH} \]
\[ V_1 = 5 + 2 - 1 + 1 = 7 \]
\[ V_2 = 5 + 2 - 2 + 1 = 6 \]
\[ V_3 = 5 - 1 - 2 - 2 = 0 \]
\[ V_4 = 5 + 1 + 1 - 2 = 5 \]
\[ L x 4 = 7 + 6 + 0 + 5 = 18 \ \text{mH} \]

10) Place correct dot convention below 2 coils

![Diagram of a transformer circuit with labeled parts](image)

11) Write KVL governing the circuit below & find resonance frequency \( f_0 \) if

\[ R = 10 \ \Omega, \ L_1 = L_2 = 10 \ \text{mH}, \ M = 2 \ \text{mH}, \ C = 0.1 \ \text{mF} \]

\[ -V + iR + (L_1 + L_2 - 2M) \frac{di}{dt} + \frac{1}{C} \sin \theta = 0 \]

KVL (Exact Form)

\[ -V + IR + j \omega (L_1 + L_2 - 2M) I - \frac{1}{\omega C} I = 0 \]

KVL (S-S AC Form)

\[ f_0 = \frac{1}{2\pi \sqrt{(L_1 + L_2 - 2M)C}} \]
12) KVL

\[-30\cos t + (1+j2 + j4 - j2) I - (j1 \times 2) I = 0\]

\[ (1+j2) I = 30 \angle 0^\circ \]

\[ I = \frac{30 \angle 0^\circ}{1+j2} \]

\[ V_0 = I [Jf4 - j2] - j1 [I] = j I \]

\[ \frac{30 \angle 90^\circ}{1+j2} = 13.041 \angle 26.56^\circ \]

\[ V_0 = 13.041 \cos (2t + 26.56) \]

13) Find power lost in 1Ω resistor.

\[-[50\cos t] + (2+j2) I_1 - j1 [I_2] = 0 \quad \text{--- (1)}\]

\[ (1+j4 - j2) I_2 - j1 [I_2] = 0 \quad \text{--- (2)}\]

Solving for \( I_2 = \)

\[ \frac{I_2 \text{ (RMS)}}{J2} \]

\[ P_{12} = I_2^2 \times R \text{ (RMS)} \]
Write Mesh eq's governing the circuit below.

\[ -[1020^\circ] + (1 + j9) I_1 - j5 I_2 + j2 [I_1 - I_2] - j3 [I_2] + j2 [I_1] = 0 \]

\[ j5 [I_2 - I_1] + j6 I_2 - j2 I_2 - j2 [I_1] - j3 [I_2] = 0 \]

Write mesh eq's.
**Ideal Transformer (Circuit Concept)**

It can store/transfer ideally electrical energy.

\[ L_1 \rightarrow \alpha \quad M \rightarrow \alpha \]

\[ L_2 \rightarrow \beta \]

\[ K = 1 \quad \rightarrow \text{co. eff. of coupling.} \]

\[ \rightarrow \text{No losses} \]

\[ \rightarrow 100\% \text{ } N \]

\[ L_1 : L_2 : M = N_1^2 : N_2^2 : N_1 N_2 \]

**Turns Ratio:**
\[ \frac{N_1}{N_2} = \sqrt{\frac{L_1}{L_2}} = \frac{L_1}{M} = \frac{M}{L_2} \]

**T-equivalent representation of Ideal transformer (Circuit concept)**

**(A) Mutually Adding**

\[ V_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \]

\[ V_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \]

\[ V_1 = (L_1 - M) \frac{di_1}{dt} + M \left[ \frac{di_1}{dt} + \frac{di_2}{dt} \right] \]

\[ V_2 = \]
(b) Mutually opposing

\[ i_1 \rightarrow i_2 \rightarrow L_1 \rightarrow L_2 \rightarrow (L_1 + M) \rightarrow (L_2 + M) \]

\[ I_1 \rightarrow I_2 \rightarrow -M \]

**Ideal Transformer Machine Concept**

\[ V_1 \rightarrow E_1 \rightarrow N_1: N_2 \rightarrow I_2 \rightarrow V_2 \rightarrow (Load) \]

**But,**

\[ a = \frac{1}{K} = \frac{N_1}{N_2} = \frac{E_1}{E_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1} \]

\[ \rightarrow \text{Turns ratio} \]

**K**

\[ K = \frac{N_2}{N_1} = \frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{I_2}{I_1} \]

\[ \rightarrow \text{Voltage transformation ratio} \]
Concept of Reflecting Primary or Secondary circuit parameters to simplify the TGF as a single circuit.

\[
R_2' = \frac{R_2}{K^2} \quad (k = \frac{N_2}{N_1})
\]

Secondary resistance reflected to primary.

Also:

\[
X_2' = \frac{X_2}{K^2}, \quad Z_2' = \frac{Z_2}{K^2}
\]

Primary resistance reflected to secondary.

1) What is the value of ‘n’ for which max. power is transferred to the load.

\[
R_2' = \frac{R_2}{K^2} = \frac{R_2}{C' \ln^2}
\]

\[
R_1 = \frac{4.5 \Omega}{[R_1]}
\]

\[
R_2 = \frac{5 \Omega}{[\text{load}]} = 5n^2
\]

For \( P_{\text{max}} \):

\[
R_L = R_S \Rightarrow 5n^2 = 4.5 \Rightarrow n^2 = 0.9 \Rightarrow n = 3
\]

2) \( R_1 + R_2' = 3 + \frac{4}{C' \ln^2} = 4.2 \)

\[
\frac{4}{I_1} = \frac{V_S}{R_1} = \frac{2000}{4} = 500 \, \text{V}
\]

\[
k = \frac{I_1}{I_2} = \frac{2}{1}
\]

\[
I_1 = 7, \quad I_2 = 7 \\
I_2 = \frac{I_2}{2} = \frac{500}{2} = 250 \, \text{A}
\]
3) \[ \text{Req} = \frac{14 + 36}{2} = 50 \, \Omega \]

4) \[ \text{M} \]

\[ \text{Req} = \text{?} \] Can be solved by both methods: mutually adding mutually opposing

5) \[ \text{Find } V_0 \]

(a) \[ V_0 \left[ 1 - \frac{M}{L_1} \right] \]

(b) \[ V_0 \left[ 1 - \frac{M}{L_2} \right] \]

(c) \[ V_0 \left[ 1 - \frac{L_1}{M} \right] \]

(d) \[ V_0 \left[ 1 - \frac{L_2}{M} \right] \]

\[ V_0 = V_s - M \frac{d i_1}{d t} = L_1 \phi V_s - M \left( \frac{V_s}{L_1} \right) \]

\[ V_0 = V_s \left[ 1 - \frac{M}{L_1} \right] \]

6) \[ Z_T = Z_1 + Z_2 \]

\[ = (2 + j4) + \frac{(4 - j4)}{(2j)^2} = (3 + j) \, \Omega \]
4) \( \text{Eq} = L_1 - M + \left[ \frac{M_1 (L_2 - M)}{M + L_2 - M} \right] \)

\[ = L_1 - M + \frac{M_1 L_2 - M^2}{L_2} \]

\[ = L_1 - \frac{M^2}{L_2} \]

\[ \Rightarrow \text{Dot convention will not affect circuit performance.} \]
\[ \text{It's just used for analysis purpose.} \]

5) What is the value of \( k \) for which the branch undergoes resonance?

For resonance: \( X_L = X_C \)

\[ X_{eq} = \omega L_{eq} \]

\[ = \omega \left[ L_1 + i_2 - 2M \right] \]

\[ = \omega L_1 + \omega L_2 - 2 \omega k \sqrt{L_1 L_2} \]

\[ = \omega L_1 + \omega L_2 - 2 k \sqrt{(\omega L_1)(\omega L_2)} \]

\[ = 8 + 8 - 2k \sqrt{8 \times 8} \]

\[ = 16 - 16k \]
$X_L = X_C$

$16 - 16k = 12 \Rightarrow 16k = 4$

$k = 0.25$
Electrical

Transients

- Transients are considered as sudden changes in the state of a circuit or system which are indicated by the switch operation.
- Transients occur in any electrical circuit or system as most of the system's are adaptable for quick sudden changes in time.
- Transients are also considered as an argument how the input command to output response as a new changes from previous steady state to next steady state w.r.t the state variables.
  \[ I, L, V, c \]
- Though transients occur for very short duration in time, their impact is huge in determining the entire steady state response.
- Though customers look into steady state performance of an electrical device or system, the designers are more interested in transient state performance as they give the critical design specification values.
**State Variables:**

These are the critical parameters that must be observed to determine the transient state solution in any net.

(a) In a capacitor:  
\[ \frac{\text{d}V}{\text{d}t} = \frac{\text{d}i}{C} \]

(KCL \(\rightarrow\) Nodeal) \(\rightarrow\) Voltage across capacitor is correct S.V.

(b) In an inductor:  
\[ \frac{\text{d}V}{\text{d}t} = L \frac{\text{d}i}{\text{d}t} \]

(KVL \(\rightarrow\) Mesh) \(\rightarrow\) Current through inductor is correct S.V.

\[ \rightarrow \] The response of any net/rlw when a source is present is called as forced response.

This response is independent to the nature of passive elements & it can be different for diff. types of i/p's.

Example: DC ckt Analysis, AC ckt Analysis, etc.

The response of any ckt or rlw without any source is called as Natural response.

This response is independent to i/p but purely depends upon the nature of passive elements & it is always unique determined by the characteristic eq.
governing the n/w.

This source free response is possible, provided the n/w has initial stored energy (inductor stores energy in electromagnetic form: \( V = L \dot{i} \rightarrow I_0 \).

capacitor stores energy in electrostatic form: \( q = CV \rightarrow V_0 \).

\[
\text{[Complete resp.]} = \text{[Forced resp.]} + \text{[Natural resp.]} \\
\downarrow \quad \text{Zero state response} \quad \downarrow \text{Zero i/p response}
\]

Any n/w or sys. reaches steady state after overcoming transient state. So in this chapter, as we are determining state response, all the solutions of voltage, current, power, energy, etc are w.r.t time.

**Initial Conditions:**

These are critical values of volty across capacitor & current through inductors from the previous steady state of a n/w which are specifically indicated as:

- \( t = 0^- \rightarrow \) instant just before slow operation (steady state before slow operation)
- \( t = 0^+ \rightarrow \) instant just after slow operation (TRANSIENT state operation)
\[ t \to \infty \Rightarrow \text{Steady state after slow operation} \]

\textbf{Order of ckt or n/w :—}

The no. of energy storage components available in distributed form in any ckt represents its order.

\textit{Ex:—} \( R-L, R-C \rightarrow 1^\text{st} \) order n/w

\( R-L-C, L-L-R, C-C-R, L-C \rightarrow 2^\text{nd} \) order n/w

\textbf{Note:—}

1. A capacitor will never allow sudden change in voltage across it.
   \[ V_c(0^-) = V_c(0) = V_c(0^+) \]

2. A current will inductor will never allow sudden change in current through it.
   \[ i_L(0^-) = i_L(0) = i_L(0^+) \]

\textbf{Behaviour of passive elements in transient state in comparison to Steady state :—}

If analysing of n/w as \( t \to 0^+ \), is considered as transient solution then, analysing the same n/w as \( s \to \infty \) (Steady state freq. resp.) is also considered as Transient solution. Hence L.T. are powerful tools to analyse the n/w during TRANSIENT STATE.
<table>
<thead>
<tr>
<th>Element</th>
<th>D.C. ss</th>
<th>A.C. ss</th>
<th>Transient State</th>
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<tr>
<td></td>
<td>(s = 0)</td>
<td>(s = jω)</td>
<td>(t → 0+)</td>
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<tr>
<th>R</th>
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<th>Z_R = R</th>
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<th>S.C.</th>
<th>V_lags</th>
<th>O.C.</th>
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<td>V_lag</td>
<td>Z_L = jωL</td>
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<td>ϕ = 90°</td>
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<td>V_lead</td>
<td>Z_C = \frac{1}{jωC}</td>
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<td></td>
<td></td>
<td></td>
<td>ϕ = 90°</td>
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**[I] Source Free 1st Order:**

(a) **R-C circuit:**

```
\[ V(t) = V_0 \exp\left(-\frac{t}{RC}\right) \]
```

Let \( V(0) = V_0 \)

\[ i_L + i_C = 0 \]

\[ c_\frac{dV}{dt} + \frac{V}{R} = 0 \]

\[ c_\frac{dV}{dt} = -\frac{V}{R} \quad \Rightarrow \quad \int \frac{dV}{V} = \int \frac{-dt}{RC} \]

\[ \ln[V] = -\frac{t}{RC} + \ln[A] \]

\[ \ln\left[\frac{V}{A}\right] = -\frac{t}{RC} \quad \Rightarrow \quad V = A e^{-\frac{t}{RC}} \]

But at \( t=0, V = V_0 \) then \( A = V_0 \)

\[ V(t) = V_0 e^{-\frac{t}{RC}} \]

\( T = RC \rightarrow \text{Time constant of R-C network} \)
\[ V(t=0) \rightarrow V_0 \]
\[ V(t=\tau) = e^{-\tau} V_0 = 0.367 V_0 \]
\[ V(t=2\tau) = e^{-2\tau} V_0 = 0.135 V_0 \]
\[ V(t=3\tau) = e^{-3\tau} V_0 = 0.049 V_0 \]
\[ V(t=4\tau) = e^{-4\tau} V_0 = 0.018 V_0 \]
\[ V(t=5\tau) = e^{-5\tau} V_0 = 0.006 V_0 \]

\(0 \leq t \leq 4\tau\) Transient state

\(t \geq 5\tau\) Steady state

Expression of current through capacitor,
\[ i_c = C \frac{dV}{dt} = C \frac{d}{dt} \left( V_0 e^{-\frac{t}{\tau}} \right) = C V_0 e^{-\frac{t}{\tau}} \left( -\frac{1}{\tau} \right) \]
\[ i_c = -\frac{V_0}{R} e^{-\frac{t}{\tau}} \]

Expression of power dissipated through \(R\)
\[ P_R(t) = \frac{[V(t)]^2}{R} = \frac{[V_0 e^{-\frac{t}{\tau}}]^2}{R} \]
\[ = \frac{V_0^2}{R} e^{-\frac{t}{\tau(2)}} \]
Expression for energy decay in the capacitor:

\[ E_c(t) = \frac{1}{2} C [V(t)]^2 \]

\[ = \frac{1}{2} C V_0^2 e^{-t/\tau} \]

Expression for charge:

\[ q(t) = C V(t) \]

\[ q = C V_0 e^{-t/\tau} \]

(b) \( R-L \) circuit:

\[ \begin{align*}
\begin{array}{c}
\text{Let } i(0) = I_0 \\
\text{VA KVL: } V_R + V_L = 0 \\
iR + L \frac{di}{dt} = 0
\end{array}
\end{align*} \]

\[ \therefore \frac{di}{dt} = -\frac{I_0}{R} \]

\[ \int \frac{di}{i} = -\frac{R}{L} dt \]

\[ \ln|i| = -\frac{R}{L} t + \ln[A] \]

\[ \Rightarrow \ln\left(\frac{i}{A}\right) = -\frac{R}{L} t \]

\[ i = Ae^{-\frac{R}{L} t} \]

At \( t=0 \), \( i = I_0 \) \hspace{1cm} \text{then} \hspace{1cm} A = I_0 \]

\[ i(t) = I_0 e^{-\frac{R}{L} t} = I_0 e^{-t/\tau} \]

\[ I(t) = I_0 e^{-t/\tau} \hspace{1cm} \tau = \frac{L}{R} \hspace{1cm} \text{time constant of } R-L \text{ circuit} \]
Expression of voltage across inductor:
\[ V_L = \frac{L}{d} \frac{di}{dt} = L \frac{d}{dt} (I_o e^{-t/T}) \]
\[ = -I_o R e^{-t/T} \]

\[ V_R + V_L = 0 \]
\[ V_R(t) = -V_L(t) = I_o R e^{-t/T} \]

Expression for power dissipated in the resistor:
\[ P_R = \frac{1}{2} (i(t))^2 R = \left( I_o e^{-t/T} \right)^2 R \]
\[ \Rightarrow P_R(t) = I_o^2 R e^{-2t/T} \ W \]

Expression for energy decay in inductor:
\[ E_L(t) = \frac{1}{2} L \left[ i(t) \right]^2 = \frac{1}{2} L \left( I_o e^{-t/T} \right)^2 \]
\[ = \frac{1}{2} L I_o^2 e^{-2t/T} \ J \]
Expression for Flux linkages
\[ \Psi(t) = L \dot{i}(t) = L I_0 e^{-t/T} \text{ we-T}. \]

Here the power dissipated by the resistor & energy decay in inductor or capacitor is 2 times faster than current or voltage respectively.

Time Constant :- It is the time taken by the resp to reach 36.7% of its initial value or it is also defined as time taken by the resp to reach 63.4% to its final value.

The unit of Time const. is : seconds

Then, w.r.t the units only :-

\[ \text{sec} = \text{sec} \]

\[ T_i = T_v \]

\[ \frac{L}{R} = RC \]

\[ \Rightarrow \text{Unit of } \frac{L}{RC} = \Omega \]

Unit of \( R^2C = H \)

Units of \( \frac{L}{R^2} = F \)

Units of \( \frac{RC}{L} = \Omega \)
\[ \text{Units of } \frac{L}{C} = (\Omega)^2 \]
\[ \text{Units of } \sqrt{\frac{L}{C}} = \Omega \]
\[ \text{Units of } \frac{R^2 C}{L} = 1 \]

Time constant determines the time with which the state variable responds.

1) Determine the time constant of each shown below.

\[ T = \text{Req} \times \text{Cap} \]
\[ = 1 \text{k} \times 5 \mu = 5 \text{msec} \]

\[ T = 4 \times 2 = 8 \text{ \mu sec} \]

\[ T = 2 \times 2 = 4 \text{ \mu sec} \]

\[ T = \frac{1}{\frac{L}{R}} = \frac{2}{3} \text{ \mu sec} \]
6.
\[ L_{eq} = 6 \, \mu F \]
\[ R = 3 \, \Omega \]
\[ t = 2 \, \mu s \] sec.

7.
\[ R_{eq} = 6 \Omega / 3 \Omega = 2 \, \Omega \]
\[ t = R_{eq} \cdot C = 2 \cdot 4 \, \mu s = 8 \, \mu s \] sec.

8.
\[ t = \frac{22 \, \mu H}{11} = 2 \, \mu s \] sec.

9.
\[ R_{eq} = \frac{3}{11} \left[ \frac{3 + 3}{4} \right] = \frac{6}{11} \times \frac{15}{4} = \frac{45}{4} \times \frac{1}{2} = \frac{45}{3} \, \Omega \]
\[ t = \frac{4 \, \mu s}{R_{eq}} = \frac{5}{3} \times 9 = 15 \, \mu s \] sec.

10.
\[ \]
Here the inductors & capacitors cannot be lumped together. This is a second order L-L-R circuit.

This circuit has multiple time constants in multiple segments & the solution to these state variables can be determined by solving simultaneous differential eqns or in a simple way by using Laplace Transforms (L-T).

11. If \( V(0) = 15 \text{ V} \) find complete expression for \( i_x \).

This is a source-free 1st order R-C net.

\[ V(t) = V_0 e^{-t/\tau} \]

\( V_0 = 15 \text{ V} \) (given)

\( \tau = R \cdot C = (5/120) \cdot 0.1 = 0.04 \text{ sec} \)

\[ V(t) = 15 \cdot e^{-t/0.4} = 15 e^{-2.5t} \text{ V} \]

But

\[ i_x(t) = \frac{V(t)}{12 + 8} = 0.75 e^{-2.5t} \text{ A} \]

12. Find complete expression for \( i \).

This is a source-free, 1st order R-L net.

\[ S.V \to 'i' \]
\[ i(t) = I_0 e^{-t/T} \]

\[ T = \frac{2}{R} = \frac{V_i}{2} \]

\[ i(t) = \frac{1}{2} \text{ sec} \]

\[ i(t) = 10 e^{-t/4} \]

\[ = 10 e^{-4t} \text{ A} \]

- Determine the complete expression for \( V \& \)

2) Determine the energy stored in the capacitor

until 2nd second.

This is a source free 1st order R-C n/w
(delayed)

State variable \( \rightarrow V \)

\( 0 < t \leq 2 \) \rightarrow Steady state only.

\( t > 2 \) \rightarrow Transient solution.

\[ V = V_o e^{-t/T} \]

Here \( V_o \rightarrow \text{means} \rightarrow V(2^-) \) \( \text{C from previous state} \)

\( V_o = 5 \text{ V} \)

\( \therefore \)

For \( t > 2 \) \[ T = R \cdot C = \frac{1}{3} \times 3 = 1 \text{ sec} \]

\[ V(t) = 5 e^{-\frac{t-2}{1}} \]
The complete expression:

\[ V(t) = \begin{cases} 
5V & ; 0 \leq t \leq 2 \\
5e^{-(t-2)} & ; t > 2 
\end{cases} \]

\[ E_c(t=3) = \frac{1}{2} C(V)^2 \quad \text{as } t=3 \]

Now, \[ V(t=3) = 5e^{-(3-2)} \]
\[ = 1.84 \text{V} \]
\[ E_c = \frac{1}{2} \times \frac{1}{3} \times (1.84)^2 \]
\[ = 0.56 \text{J} \]

3) Classical Method

\[ \frac{1}{L} \frac{d}{dt} \left[ L i_1 + 2i \right] = 0 \quad \text{(1)} \]
\[ = \frac{1}{L} L i + 3i + 2 \frac{d}{dt} i = 0 \quad \text{(2)} \]
\[ 5i = -6i \]
\[ \frac{1}{L} \frac{d}{dt} i + 2i + 2 \left( \frac{-5}{6} \right) i = 0 \]

\[ \int \frac{1}{L} \frac{d}{dt} i = \int \frac{-i}{3} \]
\[ \Rightarrow \ln[i] = -\frac{2}{3} t + \ln[A] \]
\[ i = Ae^{-\frac{2}{3} t} \]

At \( t=0 \), \( i=10 \)
\[ \Rightarrow A = 10 \]

So \( A = 10 \)

\[ i(t) = 10e^{-\frac{2}{3} t} \]
$i(t) = I_0 e^{-t/T}$

$I_0 = 10 \text{ A (given)}$

$T = \frac{L}{R_{eq}}$

$\frac{1}{2} \times \frac{1}{2} = -\frac{3}{4} \text{ sec}$

$\Rightarrow R = \frac{1}{3} \Omega$

$-1 + \frac{3V_T}{2} + \frac{V_T + 3}{4} = 0$

$\Rightarrow 3V_T = 1$

$\Rightarrow V_T = \frac{1}{3}$

$\Rightarrow R = \frac{1}{3} \Omega$

$\Rightarrow$ Such circuits are conceptual & not practical.

5) $20V \quad i(t) = 0$

$\frac{1}{16} \quad \frac{1}{2} \quad \frac{1}{3} \quad i(t)$

Find der complete expression for $i(t)$.
This is a 1st order, source free R-C circuit.

\[ \text{S.V} \rightarrow \text{V} \]

\[ t = 0^+ \]

\[ KV = 20 + i + u_i = 0 \]

\[ i = 4 \]

\[ KV = -20 + i + V(0^-) = 0 \]

\[ \therefore V(0^-) = 16 \text{ V} = V_0 \]

\[ T, t > 0 \]

\[ S_0, \quad i_T = \frac{1}{8} \]

\[ R_{eq} = \frac{1}{i_T} = \frac{1}{18} = 8.5 \Omega \]

\[ T = R_{eq} \cdot C = 8 \times \frac{1}{16} = \frac{1}{2} \text{ sec} \]

\[ V(t) = 16 e^{-2t} \text{ V.} \]

\[ \therefore i(t) = \frac{V(t)}{1+3} = 4 e^{-2t} \text{ A} \]

6) Find \( \frac{dv}{di} \)

Find complete expression for \( v \) & \( i \)

\[ 20u(t) \]

\[ 2 \text{ H} \]

\[ \frac{1}{2} \text{ F} \]

\[ \frac{R}{L} \]

\[ \frac{R}{C} \]
\[ i(t) = \frac{V}{r} = \frac{1}{2} \frac{v_i}{r} = \frac{1}{4} \text{ sec} \]

(a) \[ \frac{V(t)}{I_i} = \frac{1}{\mu L} \]

(b) \[ I(t) = I_0 e^{-t/T} \]

\[ v(t) = 20 e^{-2t} \]

\[ i(t) = 10 e^{-2t} \text{ A} \]

The current is decaying faster than voltage.

[II] Step Response of 1st order ckt:

(a) Series R-C ckt:

\[ V(t) = V_{ss}(t) + V_{tr}(t) \]

\[ V(t) = V(\infty) + [V(0) - V(\infty)] e^{-t/T} \]

[Diagram showing step response]

Case 1 with initial condition given:

Let \( V(0) = V_0 \)

\[ V(t) = V_s + [V_0 + V_0] e^{-t/T} \]
Case 2 without initial condition.

Let \( V(0) = 0 \)

\[ V(t) = V_s \left[ 1 - e^{-t/T} \right] \]

Expression for \( i_c \):

\[ i_c(t) = C \frac{d}{dt} \left[ V_s (1 - e^{-t/T}) \right] \]

\[ = C V_s \left[ 0 - e^{-t/T} \times (-1/T) \right] \]

\[ i_c(t) = \frac{V_s}{R} e^{-t/T} \]

Expression for Power:

\[ P_c(t) = V_c(t) \cdot i_c(t) \]

\[ = \frac{V_s^2}{R} \left[ e^{-t/T} - e^{-2t/T} \right] \]

at \( t = 0 \) \( \Rightarrow P = 0 \)

at \( t \to \infty \) \( \Rightarrow P = 0 \)

Expression for Energy

\[ E_c = \int_0^\infty P_c \, dt = \frac{1}{2} C [V_s]^2 \]

\[ \Rightarrow i_R(t) = \frac{V_s}{R} e^{-t/T} \]

\[ \Rightarrow \text{KVL} \quad -V_s + V_R + V_c = 0 \]

\[ V_R = V_s - [V_s (1 - e^{-t/T})] \]

\[ V_R(t) = V_s e^{-t/T} \]
\[ P_R(t) = V_R(t) \cdot i_R(t) = \frac{V_0^2}{R} e^{-2t/T} \]

\[ E_R(t) = \int_0^t P_R \, dt = \frac{1}{2} CV_s^2 \]

\[ \% \, Efficiency = \frac{\text{Output Power}}{\text{Input Power}} \times 100 \% = \frac{E_c}{E_R + E_c} \times 100 \% = \frac{\frac{1}{2} CV_s^2}{\frac{1}{2} CV_s^2 + \frac{1}{2} CV_s^2} \times 100 \% = 50 \% \]

\[ \therefore \, \eta = 50 \% \]

*Note* (Parallel R-C circuit)

\[ v(t) = I_{BR} + [V_0 - I_{SR}] e^{-t/\tau} \quad \tau = RC \]

Here we cannot connect a voltage source across capacitor as it will cause sudden change of voltage across capacitor which will result in large current which may damage the circuit.

The circuit behavior cannot be determined at \( t = 0^+ \)
(b) \( R-L \) circuit:

\[
i(t) = i_{ss}(t) + i_{tr}(t)
\]

\[
i(t) = I(0) + [I(0) - I(w)]e^{-t/\tau}
\]

\[I(0) \rightarrow \text{current through inductor before switch operation & steady state.}\]

\[I(w) \rightarrow \text{current through inductor after switch operation & steady state.}\]

\[I = \frac{L}{R}\]

**Case 1** with initial condition given:

\[
i_L(t) = \frac{V_S}{R} + \left[I_0 - \frac{V_S}{R}\right]e^{-t/\tau}
\]

**Case 2** without initial condition:

\[I(0) = 0\]

\[
i_L(t) = \frac{V_S}{R} \left[1 - e^{-t/\tau}\right]
\]

Expression for voltage across inductor:

\[
VL = \frac{V_S}{R} \left[1 - e^{-t/\tau}\right]
\]

\[
= \frac{L}{R} \left[V_S - e^{-t/\tau} \times \frac{1}{\tau}\right]
\]

\[
VL(t) = V_S e^{-t/\tau}
\]
\[ P_L(t) = V_L(t) \cdot i_L(t) = \frac{V_S^2}{R} \left[ e^{-t/T} - e^{-2t/\tau} \right] \]

\[ E_L(t) = \int P_L(t) \, dt = \frac{1}{2} L \left[ i(t) \right]^2 = \frac{1}{2} L \left[ \frac{V_S}{R} \right]^2 I \]

\[ i_R(t) = \frac{V_S}{R} \left[ 1 - e^{-t/\tau} \right] \]

\[ V_R(t) = V_S - V_L(t) = V_S - V_S \cdot e^{-t/T} = V_S \left[ 1 - e^{-t/T} \right] \]

\[ P_R(t) = V_R \cdot i_R = \frac{V_S^2}{R} \left[ 1 - e^{-t/T} \right]^2 \]

\[ E_R(t) = \int P_R(t) \, dt = \alpha I \]

\[ \% \, N = \frac{E_L}{E_L + E_R} \times 100 \% \]

\[ = \frac{\frac{1}{2} L \left( \frac{V_S}{R} \right)^2}{\frac{1}{2} L \left( \frac{V_S}{R} \right)^2 + \alpha} \times 100 \% \]

\% \, N = 0 \%

Note (Parallel R-L circuit)

\[ i(t) = i_S + \left[ I_S - i_S \right] e^{-t/T} \]

\[ T = \frac{\tau}{R} \]
1) Find the complete expression for $V$.

This is the step response of 1st order R-C network.

$s.V \rightarrow V$

$$V(t) = V(\infty) + [V(0) - V(\infty)] e^{-t/T}$$

$\Rightarrow V(0) = ?$

$\Rightarrow V(\infty) = ?$

$T = \frac{1}{2} \text{ sec}$

$5 \times 0.1 = 0.5 \Omega$

$V(\infty) = 5 \times 2 = 10V$

$V(t) = 10 + [5-10] e^{-t/0.5} = 10 - 5 e^{-2t}$

2) Find complete expression for $I$.

This is step response of 1st order R-L network.

$s.V \rightarrow I$

$$i(t) = I(\infty) + [I(0) - I(\infty)] e^{-t/T}$$
\( i(0) = \frac{10}{2} = 5 \, \text{A} \)
\( i(t) = \frac{10}{5} = 2 \, \text{A} \)
\( I = \frac{L}{R} = \frac{\sqrt{2}}{5} = \frac{1}{10} \, \text{sec} \)
\( i(t) = 2 + 3 \, e^{-10t} \, \text{A} \)

3) Find the complete expression for \( i(t) \)

This is a 1st order, step response R-L circuit.

\[ S.V \rightarrow I \]
\[ i(t) = i(\infty) + [i(\infty) - i(0)] \, e^{-t/T} \]

\[ i(0) = \frac{50}{50 + 10} \times \frac{30 \times (50 + 0)}{50 \times 10} = 6 \, \text{A} \]

\[ i(\infty) = \frac{260 \times 20}{30 \times 10} = 8 \, \text{A} \]

\[ T = \frac{\sqrt{2}}{30\sqrt{2}/30} = \frac{1}{12} \, \text{sec} \]

\[ i(t) = 8 - 2 \, e^{-12t} \, \text{A} \]

4) Find the complete expression for \( i(t) \) and the energy stored in the inductor up to 3.25 sec.

This is a 1st order, step response, R-L circuit (delayed)

\[ S.V \rightarrow i \]
\(0 < t \leq 3 \quad \rightarrow \quad \text{Steady State}\)

\[i = 3 \text{ A}\]

\(t > 3 \quad \rightarrow \quad \text{Transient Solution}\)

\[i(t) = I(\infty) + [I(0) - I(\infty)] e^{-t/\tau}\]

\(\rightarrow \text{Here } I(0) \rightarrow i(3^-) \rightarrow \text{ from previous state}\)

\[i(3^-) = 3 \text{ A}\]

\[\Rightarrow I(\infty)\]

\[10V \quad \xrightarrow{2} \quad 5 \text{ A}\]

\[\Rightarrow I, \quad t \neq 3, \quad t > 3\]

\[I = \frac{L}{R} = \frac{1/2}{2} = \frac{1}{4} \text{ sec}\]

\[i(t) = 5 + [3 - 5] e^{-(t-3)/(4)}\]

**Complete expression**

\[i(t) = \begin{cases} 3 \text{ A}, & 0 < t \leq 3 \\ 5 - 2 e^{-4(t-3)}, & t > 3 \end{cases}\]

\[E_L(t=3.25) = \frac{1}{2} L i^2\]

\[i(t=3.25) = 5 - 2 e^{-4(3.25-3)} = 4.26 \text{ A}\]

\[E_L = \frac{1}{2} \times \frac{1}{2} \times (4.26)^2 = 4.5245 \text{ J}\]
5) Find the complete expression of \( V \)

This is Step response, 1st order R-C etc.

\( S.V \rightarrow V \)

0 \( \leq t \leq 1 \) (Steady state)

\[ V(t) = V(0) + (V(0) - V(\infty)) e^{-t/\tau} \]

Here \( V(0) \rightarrow V(1-) \rightarrow \text{from previous state} \)

\[ V(1-) = 5V \]

\[ \Rightarrow V(\infty) \]

\[ KU \]

\[ -5 + V(\infty) + 5 - \frac{5}{2} = 0 \]

\[ \therefore V(\infty) = \frac{5}{2}V \]

\[ \Rightarrow \frac{1}{\tau} \]

\[ \tau = RC \]

\[ = 2 \times 1 = 2 \text{sec} \]

\[ V(t) = 2.5 \left[ 1 + e^{-\frac{(t+1)}{2}} \right] \]

\( t \geq 2 \) (2nd part of T.R)

\[ V(t) = V(\infty) + (V(0) - V(\infty)) e^{-t/\tau} \]

Here \( V(0) = V(2-) \rightarrow \text{from previous state} \)
\[ V(2') = 2.5 \left[ 1 + e^{-1/2} \right] = 4 \text{ V} \]

\[ \Rightarrow V(\phi) \]

\[ \frac{(V-5) + \frac{V}{2}}{2} - 5 = 0 \]
\[ 2V = 15 \]
\[ V = \frac{15}{2} \]

\[ K \text{VL} \]
\[ -5 + V(\phi) + \frac{15}{2} = 0 \]
\[ V(\phi) = -\frac{5}{2} = -2.5 \text{ V} \]

\[ \Rightarrow I \]

\[ V(+) = -2.5 + 6.5 e^{\frac{-(t-2)}{2}} \text{ V} \]

\[ v(t) = \begin{cases} 
5 \text{ V} & ; \quad 0 \leq t \leq 1 \\
2.5 \left[ 1 + e^{\frac{-(t-1)}{2}} \right] \text{ V} & ; \quad 1 < t \leq 2 \\
-2.5 + 6.5 e^{\frac{-(t-2)}{2}} \text{ V} & ; \quad 2 < t 
\end{cases} \]

\[ T = R \cdot C = 2 \cdot 1 = 2 \text{ sec} \]
6) Find the complete expression for \( i \).

\[ i(t) = 20 \left[ 1 - e^{-4t} \right] \text{ A} 

\[ \text{Initially relaxed} \]

\[ I(0) = 0 \text{ A} \]

\[ I(\infty) = 20 \text{ A} \]

\[ \text{KCL} \quad I(\infty) = 20 \text{ A} \]

\[ \Rightarrow I = \frac{L}{R_{eq}} \]

\[ \text{KVL} \quad -100 + 30i_x + 20i_x = 0 \]

\[ i_x = 2 \]

\[ \text{KVL} \quad -100 + 20 + V_{oc} = 0 \]

\[ V_{oc} = 80 \text{ V} \]

\[ R_{eq} = \frac{V_{oc}}{I_{sc}} = \frac{80}{20} = 4 \Omega \]

\[ T = \frac{L}{R} = \frac{1}{4} \text{ sec} \]

\[ \text{S-I} \]

\[ I_{sc} = I(\infty) \]

\[ \therefore I_{sc} = 20 \text{ A} \]

\[ \text{S-II} \]

\[ \text{Diagram} \]
III] Source free 2nd order circuits:
(Canonical form)

\( (C1) \) Series \( R-L-C \)

\[
\begin{align*}
\text{Dominant S.V. } &\rightarrow i \\
\text{KV1 } &\rightarrow \quad iR + L\frac{di}{dt} + \frac{1}{C}Si \; dt = 0 \\
R \frac{di}{dt} + L \frac{d^2i}{dt^2} + \frac{1}{C}i & = 0
\end{align*}
\]

\( \rightarrow \) Use Laplace \( \mathcal{L} \) (homogeneous)

\[
Ls^2I(s) + RsI(s) + \frac{I(s)}{C} = 0
\]

\[
I(s) \left[ Ls^2 + Rs + \frac{1}{C} \right] = 0
\]

\[
I(s) \left[ s^2 + \frac{R}{L}s + \frac{1}{LC} \right] = 0 \quad (\text{as} \ I(s) \neq 0)
\]

\[ s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \]

The 2 roots are: \( s_1, s_2 = \frac{-R}{L} \pm \sqrt{\left( \frac{R}{2L} \right)^2 - \frac{1}{LC}} \)

\[ s_1, s_2 = \frac{-R}{2L} \pm \sqrt{\left( \frac{R^2}{4L^2} \right) - \left( \frac{1}{LC} \right)^2} \]

Let \( \alpha = \frac{R}{2L} \) \{ Damping factor \}

\[ \omega_0 = \frac{1}{\sqrt{LC}} \] \{ Undamped natural freq. \}
\[ S_1, S_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \]

**Case 1:** If \( \alpha > \omega_0 \rightarrow \) overdamped

\[
\frac{R}{2L} > \frac{1}{\sqrt{LC}}
\]

→ Two roots are: -ve, real & unequal

Then,

\[
i(t) = A_1 e^{\alpha_1 t} + A_2 e^{\alpha_2 t}
\]

→ \( A_1, A_2 \) are arbitrary constants that can be determined from initial conditions.

**Case 2:** If \( \alpha = \omega_0 \rightarrow \) critically damped

\[
\frac{R}{2L} = \frac{1}{\sqrt{LC}}
\]

→ The 2 roots are: -ve, real & equal

Then,

\[
i(t) = e^{-\alpha t} [A_1 + A_2 t]
\]

**Case 3:** If \( \alpha < \omega_0 \rightarrow \) underdamped

\[
\frac{R}{2L} < \frac{1}{\sqrt{LC}}
\]

→ The 2 roots are: complex conjugate with -ve, real

Then,

\[
i(t) = e^{-\alpha t} [A_1 \cos \omega t + A_2 \sin \omega t]
\]
\[ w_d = \sqrt{\omega_0^2 - \alpha^2} \]

The damped freq.

(b) Parallel R-L-C:

\[
\begin{align*}
K + \frac{V}{R} + \frac{1}{L} SV \ast t + C \frac{dV}{dt} &= 0 \\
C \frac{d^2V}{dt^2} + \frac{1}{R} \frac{dV}{dt} + \frac{V}{L} &= 0
\end{align*}
\]

→ Use Laplace T/If (homogeneous)

\[
C s^2 V(s) + \frac{1}{R} s V(s) + \frac{V(s)}{L} = 0
\]

\[
V(s) \left[ s^2 + \frac{1}{RC} s + \frac{1}{LC} \right] = 0
\]

\[ s^2 + \frac{1}{RC} s + \frac{1}{LC} = 0 \]

The 2 roots are: \( s_1, s_2 = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \left(\frac{1}{2LC}\right)^2} \)

Let, \( \alpha = \frac{1}{2RC} \) damping factor

\( w_0 = \frac{1}{\sqrt{LC}} \) undamped natural freq.

\( s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - w_0^2} \)
○ **Case 1** If \( \alpha > \omega_0 \rightarrow \text{overdamped} \)

\[
\frac{1}{2RC} > \frac{1}{\sqrt{LC}}
\]

→ The two roots are -ve, real & unequal.

Then,

\[
V(t) = A_1 e^{\alpha t} + A_2 e^{\beta t}
\]

→ \( A_1, A_2 \) are arbitrary constants that can be determined from initial conditions.

○ **Case 2** If \( \alpha = \omega_0 \rightarrow \text{critically damped} \)

\[
\frac{1}{2RC} = \frac{1}{\sqrt{LC}}
\]

→ The two roots are -ve, real & equal.

Then,

\[
V(t) = e^{\alpha t} [A_1 + A_2 t]
\]

○ **Case 3** If \( \alpha < \omega_0 \rightarrow \text{underdamped} \)

\[
\frac{1}{2RC} < \frac{1}{\sqrt{LC}}
\]

→ The 2 roots are complex conjugate with -ve, real.

Then,

\[
V(t) = e^{\alpha t} \left[ A_1 \cos \omega_d t + A_2 \sin \omega_d t \right]
\]

\( \omega_d = \sqrt{\omega_0^2 - \alpha^2} \)
1) The nature of response of $i(t), \ t > 0$ is:

- (a) $UD$
- (b) $CD$
- (c) $CD$
- (d) Sinusoidal

This is 2nd order, source free, series $R-L-C$ circuit.

$$\chi = \frac{1}{2\omega_c} = \frac{1}{2 \times 1} = \frac{1}{2} \quad \Rightarrow \quad \chi < \omega_0$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times 1}} = 1$$

2) The nature of resp. of $V(t), \ t > 0$ is:

- (a) $V(t)$
- (b) $V(t)$
- (c) $V(t)$
- (d) $V(t)$

This is 2nd order, source free, parallel $R-L-C$ circuit.

$$\chi = \frac{1}{2RC} = \frac{1}{2 \times 1 \times 1} = \frac{1}{2} \quad \Rightarrow \quad \chi = \omega_0$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times 1}} = 1$$
IV] Step Response of 2nd Order sys.

(a) Series R·L·C

\[ V(t) = V_{ss}(t) + V_{tr}(t) \]

But \[ V_{ss}(t) \] is as \( t \to \infty \)

But \[ V_{tr}(t) \] depends upon the values of R·L·C

So here, \[ \alpha = \frac{R}{2L} \quad \omega_0 = \frac{1}{\sqrt{LC}} \]

So the complete solution is:

\[ V(t) = \begin{cases} 
V_s + A_1 e^{\alpha t} + A_2 e^{\frac{-\alpha}{2} t} & \text{overdamped} \ (\alpha > \omega_0) \\
V_s + e^{\omega_0 t} [A_1 + A_2 i t] & \text{critically damped} \ (\alpha = \omega_0) \\
V_s + e^{\omega_0 t} [A_1 \cos \omega_0 t + A_2 \sin \omega_0 t] & \text{underdamped} \ (\alpha < \omega_0) 
\end{cases} \]
(b) Parallel R-L-C

\[ y = R \frac{1}{s^2 + s \omega_0 + \frac{1}{L}} \]

**Dominant S.V. \( \to \ i \)**

\[ i(t) = I_{ss}(t) + I_{tr}(t) \]

**But**

\[ I_{ss}(t) \to 0 \text{ as } t \to \infty \]

**But**

\[ I_{tr}(t) \text{ depends upon the } \text{values of } R-L-C. \]

So here,

\[ \alpha = \frac{1}{2RC}, \quad \omega_0 = \frac{1}{\sqrt{LC}} \]

So, complete solution \( i(t) \),

\[ i(t) = \begin{cases} 
I_s + A_1 e^{\alpha t} + A_2 e^{\beta t} & \text{over damped (} \alpha > \omega_0 \text{)} \\
I_s + e^{-\alpha t} [A_1 + A_2 t] & \text{critically damped (} \alpha = \omega_0 \text{)} \\
I_s + e^{-\alpha t} [A_1 \cos \omega t + A_2 \sin \omega t] & \text{under damped (} \alpha < \omega_0 \text{)}
\end{cases} \]

**Note:**

In determining the solution to 2nd order circuits, we need to find the correct values of constants \( A_1 \) & \( A_2 \) through initial conditions.
In 2nd order ckt, there are 2 condts., so we require mini. 2 eqts.

V] Initial Condition Problems:
(Transient State Problems, i.e. problem at $t = 0^+$)

Equivalent repckt representation of passive elements during transient state (at $t = 0^+$):

<table>
<thead>
<tr>
<th>Element</th>
<th>Equi. ckt ($t = 0^+$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{R}{C}$</td>
<td>$\frac{R}{C}$</td>
</tr>
<tr>
<td>$L$</td>
<td></td>
</tr>
<tr>
<td>$-I_0$</td>
<td></td>
</tr>
<tr>
<td>$+V_0$</td>
<td></td>
</tr>
<tr>
<td>$-i_0$</td>
<td></td>
</tr>
</tbody>
</table>

\[ \begin{align*}
\text{At } t = 0^-, s/v \text{ was open } \\
\therefore i(0^-) = 0 \text{ A } \\
\text{At } t = 0^+, s/v \text{ was closed } \\
\text{inductor cannot allow sudden change in current.}
\end{align*} \]

KVL
\[ -100 + 10i + \left(1 + \frac{di}{dt}\right) = 0 \implies \text{exact form} \]
\[
\begin{align*}
\Delta t + \delta t &= 0^+ \\
-100 + 10 \left[ i(0^+) \right] + \mu \frac{d}{dt} i(0^+) &= 0 \\
\Rightarrow \frac{d}{dt} i(0^+) &= 100 \, k \, A/\text{sec}
\end{align*}
\]

Differentiating,
\[
10 \frac{d}{dt} i + \mu \frac{d^2 i}{dt^2} = 0
\]
\[
\Delta t + \delta t
\]
\[
10 \frac{d}{dt} i(0^+) + \mu \frac{d^2}{dt^2} i(0^+) = 0
\]
\[
\Rightarrow \frac{d^2}{dt^2} i(0^+) = -1000 \, m \, A/\text{sec}^2
\]

\[\Rightarrow \text{Suppose in this problem find complete solution.} \]
\[i(t) = I(0) + [I(0) - I(\infty)] e^{-t/\tau}\]

Here,
\[
I(0) = 0 \, A
\]
\[
I(\infty) = \frac{100}{\mu} = 10 \, A
\]
\[
\tau = \frac{L}{R} = \frac{1000}{\mu} = 10^{-4}
\]
\[
i(t) = 10 \left[ -e^{-1000t} \right] A
\]

2) Find \( i(0^+) \), \( v(0^+) \), \( \frac{di(0^+)}{dt} \), \( \frac{dv(0^+)}{dt} \); \( i(\infty) \), \( v(\infty) \)

\[
\text{Diagram}
\]
1. Past \((t=0^-)\)

\[
\begin{align*}
\frac{i(0^-)}{I} &= \frac{400}{5} = 80\, \text{A} \\
\frac{i(0^+)}{I} &= 80\, \text{A} \\
V(0^-) &= 240\, \text{V} \rightarrow V(0^+) 
\end{align*}
\]

2. Present State \((t=0^+)\)

\[
\frac{\text{d}i(0^+)}{\text{d}t} = \frac{V_L(0^+)}{L} \\
\frac{\text{d}V(0^+)}{\text{d}t} = \frac{i_c(0^+)}{C}
\]

**Nodal**

\[-80 + \frac{V}{6} + \frac{V-240}{2} + \frac{V}{3} = 0\]

\[-480 + V + 3V - 720 + 2V = 0\]

\[
V = 200\, \text{V}
\]

**KVL**

\[-400 + 160 + V_L(0^+) + 200 = 0\]

\[
V_L(0^+) = 40\, \text{V}
\]

\[
\frac{\text{d}i(0^+)}{\text{d}t} = \frac{V_L(0^+)}{L} = \frac{40}{4\, \mu\text{F}} = 10\, \text{kA/}\text{sec}
\]

\[
i_c(0^+) = \frac{V-240}{2} = \frac{(200)-240}{2} = -20
\]

\[
\frac{\text{d}V(0^+)}{\text{d}t} = \frac{i_c(0^+)}{C} = \frac{-20}{1\, \mu\text{F}} = -20\, \text{MV/}\text{sec}
\]
3) Future ($t \to \infty$)

\[ I(\infty) = \frac{400}{4} = 100 \, \text{A} \]

\[ V(\infty) = 200 \, \text{V} \]

One of these is correct solution.

4) Find $V_s(0^+)$

\[ t = 0 \]

\[ V \left[ \frac{R_1}{R_1 + R_2} \right] \]

\[ V \left[ \frac{R_2}{R_1 + R_2} \right] \]

\[ V \left[ \frac{R_1}{R_2} \right] \]

\[ V \left[ \frac{R_2}{R_1} \right] \]

\[ V \left[ \frac{R_2}{R_1} \right] \]

1) $t = 0^-$

\[ i(0^-) = \frac{V}{R_2} \, \text{A} \]

\[ V(0^-) = 0 \, \text{V} \quad V_S(0^-) = 0 \, \text{V} = V(0^+) \]

2) $t = 0^+$
2) Find \( V_i(t^+) \) \( \frac{d}{dt} i(t^+) \)

\[
V_i(t^+) = \frac{V_{s^+}}{R_2} \times R_1
\]

1) \( t^+ = t^+ \)

\[
i(0^-) = \frac{100}{4 \times 10^6} = \frac{5}{2} \text{ A}
\]

\[
V(0^-) = \frac{20 \times 100}{400} = 50 \text{ V}
\]

2) \( t^+ = t^+ \)

\[
V_L(t^+)= 0
\]

\[
V_L(t^+) = 50
\]

\[
\frac{di(t^+)}{dt} = \frac{V_L(t^+)}{L} = \frac{50}{\mu \text{H}} = 50 \text{ kA/sec}
\]
1. \( t < 0 \) \( t = 0^- \)

\( V_a(0^-) = 10 \text{V} \)
\( V_b(0^-) = 0 \text{V} \)

2. \( t > 0 \) \( t = 0^+ \)

\( V_b(0^+) = 10 \text{V} \)

**Nodal**

\[
\frac{V_a - 10}{10} + \frac{V_a}{10} + \frac{V_a - 10}{10} = 0
\]

\( V_a(0^+) = \frac{20}{3} \text{V} \)

Find \( i_1(0^+) \), \( i_2(0^+) \), \( i_3(0^+) \)
(1) \( t = 0^-
\[ i_1(0^-) = \frac{90}{6} = 15 \text{ A}; \]
\[ i_2(0^-) = 0 \text{ A}; \]
\[ i_3(0^-) = 0 \text{ A}. \]

(2) \( t = 0^+ \)
\[ i_1(0^+) = 15 \text{ A}; \]
\[ V_1(0^-) = 30 \left( \frac{2}{3} \right) = 20 \text{ V}; \]
\[ V_2(0^-) = 30 \left( \frac{1}{3} \right) = 10 \text{ V}. \]

\[ \text{Mesh} \]
\[-90 + 4i_1 + 2[i_1 - i_2] + 0 = 0 \]
\[ 2[i_2 - i_1] + 20 = 0 \]
\[ i_1 - i_3 = 15 \]
\[ \text{Solve} \]
\[ i_1(0^+) = 15 \text{ A}; \]
\[ i_2(0^+) = 5 \text{ A}; \]
\[ i_3(0^+) = 0 \text{ A}. \]

(5) \( V_L = 50 \text{ mV}; \)
\( V_s = 5 \text{ mV}; \)
\( R = 50 \Omega; \)
\( L = 1 \text{ mH}. \)
\( i(0^-) = \frac{10}{50} = 0.2 \, \text{A} \)

\( V(0^-) = 10 \, \text{V} \)

\( V_L(0+) = 20 \)

\( i_c(0+) = 0 \, \text{A} \)

\( \frac{dV_c}{dt}(0+) = \frac{0}{C} = 0 \, \text{V/sec} \)

\[ \frac{di_c}{dt} + \frac{V_L(0+)}{L} = \frac{20}{1000} = 20 \, \text{kA/sec} \]

5) **Find the complete expression for \( V(t) \)**

\[ V(t) = V_{ss}(t) + V_{tr}(t) \]

\[ V_{ss}(t) \text{ as } t \to \infty \]

\[ V_{tr}(t) \text{ depends upon values of } R, L, C \]

\[ \text{But} \]

\[ V(t) = 21 \, \text{V} \]
\[ \alpha = \frac{R}{2L} = \frac{5}{2\times 1} = 2.5 \]
\[ \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times \frac{1}{4}}} = 2 \]

\[ \alpha > \omega_0 \implies \text{overdamped} \]

So, the complete solution is of the form:
\[ v(t) = 2u_1 + A_1 e^{s_1 t} + A_2 e^{s_2 t} \]
\[ s_1 s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \]
\[ = -2.5 \pm \sqrt{(2.5)^2 - (2)^2} \]
\[ = -1.5, -1 \]

So,
\[ v(t) = 2u_1 + A_1 e^{-1.5t} + A_2 e^{-t} \tag{A} \]

1. \[ t=0^- \]
   \[ i(0^-) = \frac{2u_1}{6} = 4 \text{ A} \]
   \[ v(0^-) = 4 \times 1 = 4 \text{ V} \]

Substituting into \( \text{Eq.}\ (A) \):
\[ u_1 = 2u_1 + A_1 + A_2 \]
\[ A_1 + A_2 = -2u_1 \tag{1} \]

2. \[ t=0^+ \]
   \[ i(0^+) = \frac{4}{e} \text{ A} \]

   \[ \frac{dv(0)}{dt} = \frac{i_c(0)}{C} = \frac{4}{1/4} \]
   \[ = 16 \text{ V/sec} \]
   \[ \frac{dv}{dt} = -u_1 A_1 e^{-1.5t} - A_1 e^{-t} \]

   \[ 16 = -u_1 A_1 - A_2 \tag{2} \]
Solving \( L + 2 R \)

\[ A_1 = \frac{4}{3} \quad A_2 = -\frac{64}{3} \]

Then complete sol'n expression for \( v(t) \) is:

\[ v(t) = 24 + \frac{4}{3} e^{-4t} + \frac{64}{3} e^{-t} \]

\( \downarrow \) Find current through battery at \( t = 0^+ \) and \( t \to \infty \)

1. \( t = 0^- \)

\[ V = 10 \quad i = 0 \quad V_e = 0 \]

\( \Rightarrow \) \( Q \) is initially relaxed (i.e. zero state)

2. \( t = 0^+ \)

\[ i_{(0^+)} = \frac{10}{1} = 10 \, \text{A} \]

Now,

\[ t \to \infty \]

\[ i = \frac{10}{1} = 10 \, \text{A} \]
Find current through the battery at \( t=0^+ \) & \( t \to \infty \)

1. \( t=0^- \) (initially relaxed)

\[ V(0^-) = 0 \text{ V} \]
\[ i(0^-) = 0 \text{ A} \]

2. \( t \to \infty \)

\[ i(\infty) = 10 \text{ A} \]

Find current through the battery at \( t=0^+ \) & \( t \to \infty \)

\[ i(0+) = \frac{10}{3} \text{ A} \]

\[ i(\infty) \rightarrow \text{very high practically} \]

\[ V(0-) = 0 \text{ V} \]
\[ i(0-) = 0 \text{ A} \]

\[ V(\infty) = 0 \text{ V} \]
\[ i(\infty) \rightarrow \text{very high practically} \]
10) Determine steady state voltages across capacitor.

\[ V_2 = 80 \left( \frac{2}{5} \right) = 32 \text{ V} \]

\[ V_3 = 80 \left( \frac{2}{5} \right) = 32 \text{ V} \]

\[ V_1 = 100 \left[ \frac{40 \text{ k}}{80 \text{ k}} \right] = 8 \text{ V} \]

12) \[ I_i = 0.4e^{-t^2} \quad \text{then,} \]

\[ I_i \quad \text{S/H} \quad V_i(t) \quad \text{at} \quad t = 1 \text{ sec} \]

\[ KV_L \quad -V_i + \omega I_i + S \frac{dI_i}{dt} = 0 \]

\[ V_i = 10(0.4e^{-t^2}) + 5 \frac{d}{dt}(0.4e^{-t^2}) \]

\[ V_i(t) = 4e^{-t^2} + 4et \]

\[ L_5 \quad \text{at} \quad t = 1 \]

\[ V_i = 8 \text{ Volts} \]
Determine steady state voltages across capacitors.

\[ V_S = 40t, \ t \geq 0 \]

\[ i(t) \text{ at } t = 2 \text{ sec.} = ? \]

\[ i(t) = i(t) + i_L(t) = \frac{V}{R} + \frac{1}{L} \int_0^t V \, dt \]

\[ i(t) = \frac{40t}{10} + \frac{1}{5} \left[ \int_0^t V \, dt + \int_0^t V \, dt \right] \]

\[ = 4t + i(0) + \frac{1}{5} \int_0^t 40 \, dt \]

\[ = 4t + 5 + \frac{8t^2}{2} \]

\[ i(t) = 4t^2 + 4t + 5 \]

\[ \text{at } t = 2 \]

\[ i = 4(2)^2 + 4(2) + 5 = 29 \text{ A} \]
VI] AC Transients

AC Transients are less effective than DC Transients because:

1. Once the equipment in AC is designed for peak rated values, operating at any point other than peak value, the equipment or wire is safe.

2. Even the surges that occur can hardly travel half a cycle & get naturally suppressed.

3. Since there are natural zero voltage or current instances, we can avoid transients completely if we can operate the switch exactly at those instances of time when current or voltage is zero.

(a) R-L circuit

\[ V_s(t) = V_m \sin(\omega t) \]

\[ i(t) = i_r(t) + i_s(t) \]

\[ KVL \]

\[ -V_s + iR + L \frac{di}{dt} = 0 \]

\[ L \frac{di}{dt} + Ri = V_m \sin(\omega t) \]

→ The nature of solution for \( i(t) \) is:

\[ i(t) = \frac{V_m}{LZ} \sin(\omega t - \phi) + Ae^{-t/T} \]
\[ I = \frac{L}{R} \]

But at \( t=0 \), \( i = 0 \)

\[ i(t) = \frac{V_m}{LZ_1} \sin(\omega t - \phi) + \frac{V_m}{LZ_1} \sin \phi e^{-t/L} \]

**Note:**

1. Let \( V_S = V_m \sin(\omega t + \theta) \)

   Then, the solution for \( i(t) \) is like,

   \[ i(t) = \frac{V_m}{LZ_1} \sin(\omega t + \theta - \phi) + A e^{-t/L} \]

   But at \( t=0 \), \( i = 0 \)
\[
\begin{align*}
\theta &= \frac{V_m}{121} \sin(\theta - \phi) + A \\
A &= \frac{-V_m}{121} \sin(\theta - \phi)
\end{align*}
\]

So, complete soln is of the form

\[
i(t) = \frac{V_m}{121} \sin(wt + \theta - \phi) - \frac{V_m}{121} \sin(\theta - \phi) e^{-t/T}
\]

2. If input excitation is in cosine terms, then express the output soln for \(i(t)\) also in cosine terms.

3. For general R-L load

\[
\text{This is the instant exactly when } i = 0
\]

So we can operate the switch exactly at this instant of time, we can avoid.

**TRANSIENTS**

\[
\begin{align*}
\omega t_0 &= \phi \\
\omega t_0 &= \tan^{-1} \left( \frac{\omega L}{R} \right)
\end{align*}
\]

\[\Rightarrow\text{This is switch time, when we can avoid Transients.}\]
(b) Series R-C ckt:

\[ i(t) = i_{ss}(t) + i_{tr}(t) \]

The natural sol' of \( i(t) \) is

\[ i(t) = \frac{V_m}{1Z_1} \sin(\omega t + \phi) + A e^{-t/T} \]

where, \( 1Z_1 = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} \)
\( \phi = \tan^{-1}\left(\frac{1}{\omega RC}\right) \)

\[ T = RC \]

At \( t=0 \), \( i=0 \)

\[ 0 = \frac{V_m}{1Z_1} \sin \phi + A \Rightarrow A = -\frac{V_m}{1Z_1} \sin \phi \]

\( \Rightarrow \) so complete sol' is:

\[ i(t) = \frac{V_m}{1Z_1} \sin(\omega t + \phi) - \frac{V_m}{1Z_1} \sin \phi e^{-t/T} \]

\( i_{ss}(t) \)

\( i_{tr}(t) \)

\( \omega t \)

\( -ve \) DC offset value

\( \omega t \)
**Note:**

1. **Here:** 
   \[ V_s = V_m \sin (\omega t + \theta) \]
   
   Then:  
   \[ i(t) = \frac{V_m}{Z_l} \sin (\omega t + \theta + \phi) + A e^{-t/T} \]

   at \( t=0, i=0; \)
   \[ 0 = \frac{V_m}{Z_l} \sin (\theta + \phi) + A \implies A = -\frac{V_m}{Z_l} \sin (\theta + \phi) \]

   -> The complete soln is:
   \[ i(t) = \frac{V_m}{Z_l} \sin (\omega t + \theta + \phi) - \frac{V_m}{Z_l} \sin (\theta + \phi) e^{-t/T} \]

2. If i/p excitation is in cosine terms then express the o/p soln for i(t) also in cosine terms.

3. At what switching instant time 'to', the current in the circuit has transient free resp.
   \[ w_{to} = \phi \]
   \[ w_{to} = \tan^{-1} \left( \frac{\omega L}{R} \right) \]
   \[ 2\pi (50) t_0 = \tan^{-1} \left[ \frac{2\pi (50) (0.01)}{5} \right] \]
   \[ 180\pi t_0 = 0.56 \quad \text{rad} \]
   \[ t_0 = 1.078 \text{ msec} \]
So, here if we can operate the switch exactly at 1.78 ms from the instant when \( v_{l b} \) became zero, we can completely avoid transient.

2. In the above problem if \( V_s = V_0 \sin(\omega t - 10) \) then determine the value of \( t \) which results into transient free resp.

\[
\omega t_0 = 10^\circ = \tan^{-1}\left(\frac{\omega t_0}{10}\right)
\]

\[
2\pi \times 50 \times 10 \times \frac{\pi}{180} = \tan^{-1}\left[\frac{2\pi \times (50)(0.01)}{5}\right]
\]

\[
t_0 = 100 \pi t_0 = 0.73
\]

\[
 t_0 = 2.33 \text{ msec.}
\]

VII] Laplace Transform & Its Application to ckt Analysis:–

When to use L.T. methods in ckt analysis?

1) If determination of I is difficult

2) If order of ckt \( \geq 2 \)

3) Non-canonical form of ckt

4) Non-standard excitation

(i.e. impulse, pulse, ramp, parabolic, step, exponential, etc.)

\[ L[s(t)] \rightarrow F(s) \quad \text{[} s = j\omega \text{]} \]

\[ L\text{ complex freq} \]
\[ V(t) \leftrightarrow V(s) \]
\[ I(t) \leftrightarrow I(s) \]
\[ V \leftrightarrow \frac{V}{s} \]
\[ I \leftrightarrow \frac{I}{s} \]
\[ V_{m \sin \omega t} \leftrightarrow \frac{V_{m \omega}}{s^2 + \omega^2} \]
\[ V_{m \cos \omega t} \leftrightarrow \frac{I_{0 \omega}}{s^2 + \omega^2} \]
\[ R \leftrightarrow \frac{1}{R} \]
\[ C \leftrightarrow \frac{1}{Cs} \]
\[ L \leftrightarrow sL \]

Equivalent ckt representation of passive elements in Laplace domain:

(a) Resistor

\[
V(s) = I(s) \cdot R
\]
\[
I(s) = \frac{V(s)}{R}
\]

(b) Inductor

\[
\dot{V}(s) = I(s) \cdot sL
\]
\[
I(s) = \frac{V(s)}{sL}
\]

(c) Capacitor

\[
V(s) = \frac{I(s)}{Cs}
\]
\[
I(s) = V(s) \cdot Cs
\]

(d) Inductor with \( I_0 \)

\[
V(s) = sLI(s) - LI_0
\]
\[
I(s) = \frac{V(s)}{sL} + \frac{I_0}{s}
\]
(c) Capacitor with \( V_0 \)

\[
I(s) = Cs V(s) - CV_0
\]

\[
V(s) = \frac{I(s)}{Cs} + \frac{V_0}{s}
\]

1) If \( I_0 = 10 \text{ A} \), find the complete expression of \( I \)

(a) in Time domain

(b) in Laplace domain

(a) \( i(t) = I_0 e^{-\frac{t}{\tau}} \)

\( I_0 = 10 \text{ A} \) given.

\[
\tau = \frac{L}{R_{eq}}
\]

\[
\frac{3}{2} + 1 - V_T = 0
\]

\[
V_T = \frac{5}{2}
\]

\[
R_{eq} = \frac{V_T}{I} = \frac{5}{2} \Omega
\]

\[
I = \frac{\sqrt{2}}{5\sqrt{2}} = \frac{1}{5} \text{ sec}
\]

So, \( i(t) = 10 e^{-5t} \)

(b) KVL

\[
\frac{3}{2} I(s) + I(s) - 5 + I(s) \frac{5}{2} = 0
\]
\[ I(s) \left[ \frac{s}{2} + \frac{5}{2} \right] = 5 \]

\[ I(s) = \frac{10}{s+5} \]

\[ i(t) = \mathcal{L}^{-1} \left[ \frac{10}{s+5} \right] = 10e^{-5t} \]

2) Find \( i(t), \ t > 0 \) → This circuit is initially relaxed.

\[ \text{KVL} \]

\[ \frac{50}{s} + I(s) \left[ 2 + s + \frac{2}{s} \right] = 0 \]

\[ I(s) \left[ \frac{s^2 + 2s + 2}{s} \right] = \frac{50}{s} \]

\[ I(s) = \frac{50}{s^2 + 2s + 2} \]

\[ I(s) = \frac{50}{(s+1)^2 + (\sqrt{3})^2} \]

\[ i(t) = \mathcal{L}^{-1} [I(s)] = 50e^{-t} \sin t \cdot A \]

→ This circuit is initially relaxed.

3) Find \( v(t), \ t > 0 \) → This is initially relaxed circuit.

\[ \text{KCL} \]

\[ -\frac{2}{s} + \frac{V(s)}{1/2} + \frac{V(s)}{s} + \frac{V(s)}{1/s} = 0 \]

\[ V(s) \left[ \frac{s + 1/2 + s}{s} \right] = \frac{2}{s} \]
\[ V(s) = \left[ \frac{s^2 + 2s + 1}{s} \right] = \frac{2}{s} \]

\[ V(s) = \frac{2}{(s+1)^2} \Rightarrow V(t) = 2t e^{-t} \]

\[ \text{critically damped} \]

\[ V(s) = \frac{2}{s} \left( s + \frac{2}{s+1} \right) \]

\[ \frac{KCL}{2} + V(s) \frac{2}{s+1} + \frac{V(s)}{s/s} + \frac{V(s)}{(s+1)/2} = C \]

\[ \therefore V(s) \left[ 1 + s + \frac{s}{s+1} \right] = \frac{2}{s} \]

\[ V(s) = \frac{2}{s} \left( \frac{s+4}{s^2 + 2s + 6} \right) \]

\[ V(s) = \frac{2(s+4)}{s(s+2)(s+3)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3} \]

\[ \text{where,} \quad A = \frac{2(6+4)}{(0+2)(0+3)} = \frac{8}{6} = \frac{4}{3} \]

\[ B = \frac{2(-2+4)}{(-2)(-2+3)} = \frac{4}{-2} = -2 \]

\[ C = \frac{2(-3+4)}{(-3)(-3+2)} = \frac{-2}{3} = \frac{2}{3} \]

\[ \therefore V(s) = \frac{4/3}{s} - \frac{2}{s+2} + \frac{2/3}{s+3} \]
5. \( V(t) = \frac{4}{3} u(t) - 2e^{-2t} + \frac{2}{3} e^{-3t} \)

\[ \Rightarrow \text{over damped step resp.} \]

**Find \( i(t), t > 0 \)**

\[ \begin{align*}
\text{KVL:} & \quad -\frac{10}{5} + I(s) \left[ \frac{1 + \frac{1}{6s} + \frac{1}{3s}}{3s} \right] = 0 \\
& \quad I(s) \left[ \frac{6s + 3}{6s} \right] = \frac{10}{5} \\
& \quad I(s) = \frac{6s + 3}{6s} \\
& \quad = \frac{20}{2s + 1} \\
i(t) &= \mathcal{L}^{-1} [I(s)] = 10 \ e^{-t/2} \ \text{A} \\
\end{align*} \]

6. **Find \( i(t), t > 0 \)**

\[ \begin{align*}
v(t) &= 5V \\
i(0^-) &= \frac{5}{2.5} = 2 \ \text{A} \\
\text{KVL:} & \quad I(s) \left[ \frac{s}{2} + \frac{1}{200 \mu F s} \right] = 1 \\
& \quad I(s) \left[ \frac{s + 10^6}{100 s} \right] = 2 \\
& \quad I(s) \left[ s + \frac{10^4}{s} \right] = 2 \\
\end{align*} \]
\[ I(s) = \frac{2 \cdot 5}{s^2 + (100)^2} \]

\[ v(t) = 2 \cos 100t \quad \text{sinusoidal output} \]

Find \( V_L \):

(a) \( V_s(t) = 8(t) \)

\[ V_L(s) = V_s(s) \left[ \frac{s/2}{s/2 + 5} \right] \]

(b) \( V_s(t) = e^{-t} u(t) \)

\[ V_L(s) = V_s(s) \left[ \frac{s}{s + 10} \right] \]

\( (a) \quad V_s(t) = 8(t) \)

\[ V_s(s) = \frac{8}{s} \]

\[ \therefore \quad V_L(s) = 1 \times \frac{s}{s + 10} = \frac{s + 10 - 10}{s + 10} = 1 - \frac{10}{s + 10} \]

\[ \therefore \quad V_L(t) = 8(t) - 10e^{-10t} \]

\( (b) \quad V_s(t) = e^{-t} u(t) \)

\[ V_s(s) = \frac{1}{s + 1} \]

\[ \therefore \quad V_L(s) = \frac{1}{s + 1} \cdot \frac{s}{s + 10} = \frac{A}{s + 1} + \frac{B}{s + 10} \]

where \( A = \frac{-1}{-1 + 10} = \frac{1}{9} \)

\[ B = \frac{-10}{-10 + 1} = \frac{-10}{9} \]

\[ \therefore \quad V_L(s) = \frac{10}{9(s + 10)} - \frac{1}{9(s + 1)} \]
\[ V_e(t) = \frac{10}{q} e^{-10t} - \frac{1}{q} e^{-t} \]

8) Find the complete expression for \( V \) using Laplace transform method.

\[ i(0^-) = 4 \, \text{A} \]
\[ v(0^-) = 4 \, \text{V} \]

\[ \mathbf{\text{Laplace Transform}} \]

\[ \frac{24}{S} - 4t + \frac{4}{S} + I(s) \left[ 5 + s + \frac{4}{S} \right] = 0 \]

\[ I(s) \left[ \frac{s^2 + 5s + 14}{S} \right] = \frac{4s + 20}{S} \]

\[ I(s) = \frac{4s + 20}{s^2 + 5s + 41} \]

\[ V(s) = \frac{4}{S} + I(s) \times \frac{4}{S} \]

\[ = \frac{4}{S} + \frac{4}{S} \left( \frac{4s + 20}{s^2 + 5s + 41} \right) \]

\[ = \frac{4s^2 + 36s + 96}{S(S+1)(S+4)} \]

\[ V(s) = \frac{A}{S} + \frac{B}{S+1} + \frac{C}{S+4} \]

When, \[ A = \frac{96}{(4)(4)} = 2 \, \text{V} \]
\[ B = \frac{64 - 144 + 96}{12} = \frac{44}{3} \]

\[ C = \frac{4 - 36 + 96}{3} = \frac{64}{3} \]

\[ V(s) = \frac{24}{s} + \frac{4}{3(s+4)} + \frac{-64}{3(s+1)} \]

\[ V(t) = 24 + \frac{4}{3} e^{-4t} - \frac{64}{3} e^{-t} \]

Apply Thevenin's theorem to obtain the expression of current \( i(t) \) in \( 1 \Omega \) resistor.

Thevenin's theorem can be applied to simplify the circuit in steady state. But, this is a transient problem. So, we can convert this problem in steady state freq. domain & then apply Thevenin's theorem.

\[ Z_{TH} = \frac{1}{s} + \frac{1}{s+1} \]

\[ = \frac{s + s+1}{s+2} = \frac{s^2 + 3s + 1}{s + 2} \]
\[ V_{TH} = 2 + \left( \frac{10}{3} + \alpha \right) \left[ \frac{1}{s+2} \right] \]

\[ = 2 + \left( \frac{10 + 4\alpha}{3} \right) \left( \frac{1}{s+2} \right) \]

\[ \frac{Z_{TH}(s)}{s+1} \downarrow I(s) \quad I(s) = \frac{V_{TH}(s)}{Z_{TH}(s) + 1} \]

\[ I(s) = \frac{(2s^2 + 8s + 10)(s+2)}{(s+2)s \left[ (s^2 + 3s + 1) + s+2 \right]} \]

\[ = \frac{2s^2 + 8s + 10}{s(s^2 + 4s + 3)} \]

\[ = \frac{2s^2 + 8s + 10}{s(s+3)(s+1)} \]

\[ I(s) = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+1} \]

where,

\[ A = \frac{10}{3 \times 1} = \frac{10}{3} \]

\[ B = \frac{18 - 24 + 10}{(-3)(-2)} = \frac{4}{6} = \frac{2}{3} \]

\[ C = \frac{2 - 8 + 10}{(-1)(-2)} = \frac{4}{-2} = -2 \]

\[ I(s) = \frac{10}{3} \cdot \frac{1}{s} + \frac{2}{3} \cdot \frac{1}{s+3} - 2 \cdot \frac{1}{s+1} \]
\[ i(t) = \frac{10}{3} + \frac{2}{3} e^{-3t} - 2 e^{-t} \]

\[ \text{NETWORK FUNCTIONS \& FILTER CONCEPTS} \]

If the magnitude of \( V \) supply is kept constant, but freq. is varied, then the output is defined as the complete freq. resp. of the cir or n/w.

\[ \text{Eq.} - \text{Resonance} \]

Freq. resp. of any n/w gives its complete steady state performance which is useful in design & analysis, & synthesis of filters, antennas, & sonars, radars, etc. in communication engg.

To obtain the complete freq. resp. of the n/w, we need to build the n/w's transfer function.

\[ X(\omega) \xrightarrow{H(\omega)} Y(\omega) \quad \text{T.F.} \quad H(\omega) = \frac{Y(\omega)}{X(\omega)} \]

In n/w's, there are only 4 types of T.F. that can be defined.
(a) Voltage Gain T.F.
\[ G_v(s) = \frac{V_o(s)}{V_i(s)} \]

(b) Current Gain T.F.
\[ \alpha(s) = \frac{I_o(s)}{I_i(s)} \]

(c) Transfer Impedance
\[ Z(s) = \frac{V_o(s)}{I_i(s)} \]

(d) Transfer Admittance
\[ \gamma(s) = \frac{I_o(s)}{V_i(s)} \]

→ In general, impedance & admittance together are called as Impittance. Further, in analysing filter circuits, we generally consider V.G.T.F. : \( H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} \)

→ The new func. \( H(\omega) \) is complex quantity & so, it has magnitude & phase as \( |H(\omega)| \) & phase of \( \Phi(\omega) \).

To obtain complete freq. resp. of new T.F., we need to plot both mag. & phase as \( \omega \) is varied from 0 to \( \infty \).
1) Obtain the complete freq. resp. of the voltage gain T.F. of the circuit shown below.

\[ V_o(w) = V_i(w) \times \frac{1}{j\omega C} \]

\[ V_o(w) = V_i(w) \times \frac{1}{R + \frac{1}{j\omega C}} \]

\[ H(w) = \frac{V_o(w)}{V_i(w)} = \frac{1}{1 + j\omega RC} \]

\[ \rightarrow \text{Magnitude} \rightarrow |H(w)| = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}} \]

\[ \rightarrow \text{Phase} \rightarrow \phi(w) = -\tan^{-1}(\omega RC) \]

\[ \text{Magnitude plot} \]

\[ \text{Phase plot} \]

2) Obtain the current gain T.F. of the circuit shown and plot the location of poles & zeros in S-plane.

\[ \Sigma_i = \Sigma_i(s) \]

\[ \Sigma_o = \Sigma_o(s) \]

\[ 2 \]

\[ 1 \]

\[ 1H \]

\[ 1F \]

\[ \frac{1}{s} \]
1) Obtain the driving pt. impedance of
the n/w shown below.

\[ Z(s) = \frac{1+\frac{1}{s}}{s+\frac{1}{s}} = \frac{2+2s}{s+1} \]

2) Obtain the driving pt. impedance of n/w shown below & plot the location of poles & zero in S-plane.

\[ Z(s) = \frac{1+\frac{1}{s}}{s+\frac{1}{s}} = \frac{2+2s}{s+1} \]

\[ Z(s) = 1 + \frac{\sqrt{3}}{s} \]
\[ Z(s) = \frac{S^2 + 3S + 1}{S(S+1)} \]

\[ Z(s) = \frac{S + 1}{S} \]

\[ Z(s) = \frac{S^2 + 3S + 1}{S(S+1)} = \frac{S^2 + 3S + 1}{S+1} \]

\[ Z(s) = \frac{S + \frac{1}{S}}{S} = \frac{S^2 + 1}{S} \]

\[ Z(s) = S + \frac{1}{S} + \frac{S\sqrt{S}}{S+\frac{1}{S}} \]

\[ = S + \frac{1}{S} + \frac{S}{S^2 + 1} \]

For any R-L-C m/w & its driving pt. impedances, the poles and zeroes on left hand side of S-plane are scattered throughout the plane.
For any R-C network & its driving pt. impedance, poles & zeros are located on the -ve real axis only & are alternately placed. And, the nearest critical freq. to origin is a pole.

For any R-L network & its driving pt. impedance, all the poles & zeros are located on the -ve real axis only & are alternately placed & the nearest critical freq. to origin is a zero.

For any pure L-C network & its driving pt. impedance, all poles & zeros are located on the imaginary axis only & are alternately placed.

The pole-zero pattern of R-L driving pt. impedance is similar to R-C driving pt. admittance.

My, the pole-zero pattern of R-C driving pt. impedance is similar to R-L driving pt. admittance func.

In general, these driving pt. impedance & admittance func. are together called as Impittance func.
1) Convert into \( \Delta \) delta.

\[
Z_{ab} = 1 + s + \frac{s}{\frac{1}{s}} = s^2 + s + 1 \ \Omega
\]

\[
Z_{ac} = 1 + \frac{1}{s} + \frac{1}{\frac{1}{s}} = \frac{s^2 + s + 1}{s^2} \ \Omega
\]

\[
Z_{bc} = s + \frac{1}{s} + \frac{1}{s} = \frac{s^2 + s + 1}{s} \ \Omega
\]

2) \[ I(s) = \frac{s + 4}{(s + 2)(s + 3)} \]

\[
\text{Find the initial value of current.}
\]

\[
i(0) = \lim_{s \to 0} \frac{s(s + 4)}{(s + 2)(s + 3)} = \lim_{s \to 0} \frac{1 + 4/s}{(1 + 2/s)(1 + 3/s)} = \frac{1 + 0}{(1 + 1)(1)} = 1 \ \text{A}
\]

3) \[ i(t) = 2 + \left[u(t) - u(t-2)\right]
\]

\[= 2 + u(t) - 2(t - 2 + 2)u(t-2)\]

\[
i(s) = \frac{2}{s^2} - \frac{2 \cdot e^{-2s}}{s^2} - \frac{4 e^{-2s}}{s}
\]

4) \[ i(t) = e^{-3t} \ \text{A for} \ u(t) = u(t) \ \text{V,} \ t \geq 0
\]

\[
\text{Determine the n/w elements.}
\]

\[
\frac{v(t)}{i(t)} = \frac{u(t)}{e^{-2t}}.
\]

\[
\frac{v(s)}{i(s)} = \frac{1/s}{1/(s + 3)} = \frac{s + 3}{s} = 1 + \frac{3}{s}
\]

\[ Z(s) = 1 + \frac{1}{\frac{1}{s}} \Rightarrow R = 1 \ \Omega \ \text{? Series,} \ C = \frac{1}{3} \ F \ \text{RC n/w} \]
Note:

\[ Z(s) = k_0 + k_1 s + \frac{k_2}{s} \]

Here,
- \( k_0 \rightarrow \text{Resistance} \rightarrow \text{Value} \rightarrow k_0 \, \Omega \)
- \( k_1 \rightarrow \text{Inductance} \rightarrow \text{Value} \rightarrow k_1 \, \text{H} \)
- \( k_2 \rightarrow \text{Capacitance} \rightarrow \text{Value} \rightarrow \frac{1}{k_2} \, \text{F} \)

5) Find \( V_o = \quad \frac{s}{s+1} \) \[ V_i = 2 \sin t \]

\[ V_o = V_i \left( \frac{s}{s+1} \right) \]

But \( s = j\omega \) & here \( \omega = 1 \) \Rightarrow \( s = j \)

\[ V_o = V_i \left( \frac{j}{j+1} \right) = 2 \sin t \left[ \frac{1 \angle 90^\circ}{j2 \angle 45^\circ} \right] \]

\[ V_o = \sqrt{2} \sin (t + 45^\circ) \, \text{V} \]

**Passive Filters:**

Filters are cts which operate for a particular range of frequencies & attenuate other frequencies.

Passive filter are cts which are designed based on passive elements: R, L, C.

Active filter are cts at electronic/signal level based on OP Amp. & digital filters also perform signal processing & they are based on DSP.

Passive filter are still used at power level to control harmonics & stabilize the power feed to the load, eg: laptop chargers.